Studies on the Role of Asset Prices and Credit in the Design of Monetary and Regulatory Policy

Inaugural-Dissertation
zur Erlangung des Grades
Doctor oeconomiae publicae (Dr. oec. publ.)
an der Ludwig-Maximilians-Universität München

2008

vorgelegt von
Florian Kajuth

Referent:	Prof. Dr. Gerhard Illing
Korreferent:	Prof. Dr. Jarko Fidrmuc
Promotionsabschlussberatung:	16. Juli 2008

Acknowledgements

While writing this thesis I have received encouragement and support from a number of people. First and foremost I would like to thank my supervisor Gerhard Illing, from whom I have learnt a lot. The conversations with him helped my understanding of economic theory and practice very often much more than lectures or textbooks. Furthermore, I appreciate his support and generous funding for national and international conferences and the summer school in Eltville. Secondly, I would like to thank Jarko Fidrmuc, who kindly agreed to act as the second examiner. Moreover, I am grateful to Sven Rady, whom I was happy to win as my third examiner. In addition, financial support from the Deutsche Forschungsgemeinschaft through SFB/TR 15 is gratefully acknowledged.

During my time at the Seminar for Macroeconomics at the LMU in Munich I was privileged to meet and become friends with many enjoyable and interesting people. I would like to thank Desi Andreeva, Julia Bersch, Josef Forster, Moritz Hahn, Frank Heinemann, Uli Klüh, Katri Mikkonen, Stephan Sauer and Sebastian Watzka for their help, encouragement and many stimulating discussions over coffee. Furthermore, I would like to thank Agnès Bierprigl for her excellent support in all things related to administration, organisation and the submission process. Special mention deserves Dirk Rösing for his reliable IT support and advice. Moreover, I am grateful to Uwe Böwer, Marcus Drometer, Hannah Hörisch and Liudmyla Hvozdyk for their personal support and friendship. Finally, I would like to thank my parents, my sister and Judith for their patience, encouragement and for distracting me from my efforts with this thesis, which so often motivated me to carry on.

Contents

List of Figures

List of Tables

Chapter 1

Introduction and summary

The last couple of years have seen developments in financial markets, especially in the US, that challenge macroeconomic and monetary policy. While the era of the Great Moderation has seen a considerable decline in output and inflation volatility, at the same time asset prices have experienced large swings. Among them are the sharp rise and fall of U.S. stock prices around the turn of the millennium and the surge and subsequent slowing of U.S. house prices. The movement of European stock and house prices has been a matter of debate too (ECB, 2005a, 2005b, 2006).

Asset prices play an important role in modern economies and are of interest to policy makers for various reasons. Being inherently forward-looking asset prices can provide information about the expectations of the market regarding future productivity and inflation. Moreover, they might impact on inflation if high asset prices spill over to goods prices inflation. In addition, in the past, swings in asset prices were typically accompanied by fluctuations of credit expansion in the same direction. Moreover, asset price booms were often followed by distress in financial markets or the real economy (Borio and Lowe, 2002).

A recurring theme in this context is whether central banks should respond actively to changes in asset prices. It might be desirable to avoid boom-bust cycles in asset prices and the possible consequences for the real economy by adjusting interest rates in the face of rapid rises and falls in asset prices. There are, however, a number of problems associated with an active response to asset prices. Since it is very difficult to identify deviations from fundamentals in asset prices, it is unclear to what extent an asset price change is attributed to changes in fundamental determinants. The distinction between fundamental and non-fundamental movements, however, is crucial in designing the appropriate interest rate response. Moreover, a pre-emptive strategy to counter an asset price boom possibly at the expense of output might be hard to justify to the public. In contrast, a strategy accommodating a surge in asset prices and mitigating any real consequences from a fall might be easier to justify from an ex-ante point of view. However, this strategy has been criticised as inviting excessive risk-taking.

Within the context of increased attention to financial stability a different but related development in recent years has been the introduction of a new regulatory framework for the banking industry, the New Basel Capital Accord (Basel II). It effectively applies to banks in Europe, the U.S. and the remaining G10 countries and modifies the existing Basel Capital Accord (Basel I) in a number of dimensions. The central element is a minimum capital requirement which banks must fulfil at all times and which states that a bank has to hold at least eight percent of its risk-weighted assets in the form of capital. The requirement is aimed at increasing the stability of the banking system. The New Basel Accord introduces variable risk-weights based on borrowers' ratings to calculate risk-weighted assets. This had become necessary to avoid regulatory arbitrage due to risk-weights being assigned according to borrower category under Basel I. While the new rules certainly succeed in limiting the scope for regulatory arbitrage and in guaranteeing a floor to individual bank capitalisation there have been concerns about their macroeconomic implications, especially in terms of aggregate bank lending. One criticism that has been voiced is that capital requirements based on variable borrower ratings might unduly exacerbate business cycles. In a downswing applied risk measures increase and force banks to either take up more capital, which might be hard, or to reduce lending instead. This might lead to a worsening of a recession.

A large amount of research is being done on these issues and they are still largely unresolved. This thesis addresses three problems which are of fundamental relevance and have also featured in the recent developments in financial markets: The optimal response of monetary policy to house price movements, the moral hazard problem associated with central bank intervention in a financial crisis, and the effect of risk-based bank capital regulation on the cyclicality of aggregate bank lending. The goal is to contribute to understanding and assessing the effects of monetary and regulatory policy on macroeconomic variables like the output gap, inflation, asset prices and aggregate credit by shedding new light on these issues.

Undoubtedly these are areas of high policy relevance as the events in financial markets in 2007 and in the beginning of 2008 have demonstrated. The example of the so-called subprime crisis serves well to illustrate how the three themes of this thesis connect. Low interest rates and increasing house prices in the U.S. in recent years were associated with an expansion in home mortgage lending. This had an impact on both the build-up of debt and the ability to turn increased housing value into consumption. However, when interest rates rose and house price growth slowed, indebted homeowners increasingly defaulted on their mortgages, which in turn led to losses at banks. Moreover, non-defaulting home-owners were unable to sustain part of their consumption through home values.

One question that has received a lot of attention when house price growth started to slow is whether the Fed should adjust interest rates to a fall in house prices (see Federal Reserve Bank of Kansas City, 2007; Financial Times, 2007a). The concern was that house prices affect output and inflation, possibly with a lag, such that lowering interest rates would mitigate the consequences for the real economy. However, to ensure an appropriate policy response it is important to evaluate the exact mechanism by which house prices affect output and inflation.

Chapter 2 therefore looks at how monetary policy should optimally respond to movements in house prices. Housing is an important part of household wealth, especially in the U.S. In contrast to stock wealth it can be used to borrow against. Moreover housing equity withdrawal, a financial instrument offered by lenders in the U.S., allows homeowners to convert an increase in their home value into cash. If homeowners are liquidity constrained, which is true for a substantial part of U.S. homeowners, they are likely to expand consumption. A theoretical model is set up to capture the wealth effect on consumption arising from liquidity constrained homeowners borrowing against an increase in their housing collateral, and an aggregate demand curve is derived. The innovation is to allow for time-varying liquidity constraints. For a given state of the process of financial innovation, rising house prices alleviate liquidity constraints, and falling ones tighten them. As a result the proportion of agents who respond to movements of house prices, the output gap, expected inflation and the interest rate changes as house prices vary. An otherwise standard New Keynesian framework is used to derive the optimal interest rate rule for monetary policy. The results show that monetary policy should react to movements in house prices over and above their impact on the output gap and inflation because they affect the optimal weights on the output gap and expected inflation in the interest rate rule. The reason is that constrained agents don't consume according to a usual Euler equation but consume their current liquid assets. Therefore they react in a different way to monetary policy. Moreover, the proportion of constrained agents is determined by house prices via relaxing and tightening liquidity constraints.

In addition to the consequences for aggregate demand from falling house prices, widespread default on mortgages and subsequently on the securitised assets based on them made banks reluctant to lend to each other resulting in a liquidity shortage. This prompted the central banks in the U.S., Europe, the U.K. and elsewhere to intervene, first by injecting additional liquidity by open market operation and, in the case of the Fed, subsequently lowering the target for the Fed funds rate. This caused considerable debate among commentators and analysts, who argued that the Fed and the ECB effectively bailed-out financial market participants by injecting additional liquidity and lowering the interest rate. Critics, mainly from academia and the media, argue that providing liquidity assistance and lowering interest rates after troubles in asset markets generates moral hazard resulting in excessive risk-taking by investors, who believe the central bank will come to rescue them if things go wrong (see e.g. Financial Times, 2001). This hypothesis is known as the so-called Greenspan put because former Fed chairman Alan Greenspan lowered interest rates in response to the stock market crash in October 1987, after the crisis relating to the hedge fund Long-Term Capital Management (LTCM) in September 1998 and again in the wake of the U.S. stock market decline at the beginning of the new millennium. In contrast, others – mostly central bankers – deny that a central bank intervention in response to problems in financial markets leads to moral hazard on the part of investors (see Cecchetti, 2007; Poole, 2007) because e.g. the Fed only responds to output and inflation, and not specifically to asset prices.

Chapter 3 investigates the empirical content of the so-called Greenspan put hypothesis. Even though it has been debated among academics and in the media for almost a decade, there are only very few theoretical works and no empirical ones. Therefore we contribute to the debate by empirically testing whether the actions of the Fed in October 1987 after the Black Monday stock market crash and in September 1998 after the LTCM crisis had an impact on the stock price boom in the late 1990s. Some have argued that investors believed in an implicit guarantee by the Fed to intervene should stock prices plunge. This would result in excessive risk-taking, which pushed up stock prices. The first part of the analysis establishes that there was a statistically significant element in the valuation of stock prices that cannot be explained by the present value model for stock prices. The use of a state-space framework allows to get an inferred time-series estimate of the unexplained part of stock prices, which is commonly labelled as bubble. The second part uses variables derived from the few existing theoretical models of the Greenspan put to derive proxy indicators for moral hazard and test whether they had any significant impact on U.S. stock prices in the late 1990s. The tests are unable to confirm the hypothesis that there existed a Greenspan put option.

Widespread default on mortgages not only resulted in a liquidity shortage but also in considerable losses and write-downs at many banks. In light of the introduction of the New Basel Accord in Europe and the U.S., Goodhart (Financial Times, 2007b) points out that the losses on the part of banks might interact with the new capital requirements in a way that might exacerbate any negative consequences for the real economy. The reason is that if banks' losses erode their capital base down to the required minimum they are constrained from expanding lending. In addition, if risk-weights of borrowers rise, as is likely in a downswing, banks' regulatory capital ratio falls further and they might be forced to cut back lending exacerbating the negative consequences for the real economy even more.

Chapter 4 looks at whether the introduction of risk-based bank capital regulation à la Basel II exacerbates the cyclicality of aggregate bank lending. More specifically, the contribution is to discuss in a theoretical model the implications of time-varying risk-weights when taking into account that most banks hold capital buffers on top of the minimum capital requirement. Incorporating this stylised fact into a model with heterogeneous banks, which differ in their capital holdings, allows to work out different factors that affect the cyclicality of aggregate bank lending. Within the setup it is found that there is indeed scope for increased cyclicality of bank lending under variable risk-weights. The degree of excess cyclicality depends mainly on the sensitivity of the risk-weights with respect to changes in aggregate risk and the ease with which additional funds can be added or withdrawn from the aggregate banking balance sheet. Thus, while the New Basel Accord corrects certain failures in the original rules, it is likely to introduce another negative side-effect in the form of increased fluctuation in aggregate bank lending.

The three chapters are self-contained and can be read independently of each other.

References

Borio, C. and P. Lowe (2002): Asset prices, financial and monetary stability: exploring the nexus. Bank for International Settlements Working Paper No. 114.

Cecchetti, S. (2007): Does well-designed monetary policy encourage risk-taking? VOX at http://www.voxeu.org/index.php?q=node/99 [12 February 2008].

ECB (2005a): Asset price bubbles and monetary policy. Monthly Bulletin April 2005, 47-60.

ECB (2005b): Wealth and asset price effects on economic activity. Occasional paper series No. 29.

ECB (2006): Assessing house price developments in the Euro area. Monthly Bulletin February 2006, 55-70.

Federal Reserve Bank of Kansas City (2007): Housing, housing finance and monetary policy. A symposium sponsored by the Federal Reserve Bank of Kansas City, 30 August - 1 September, Jackson Hole, Wyoming.

Financial Times (2001): Lex - The Greenspan put. 4 January 2001.

Financial Times (2007a): Aggressive Fed action on house prices foreseen. 1 September 2007.

Financial Times (2007b): Capital, not liquidity, is the problem. 13 September 2007.

Poole, W. (2007): Market bailouts and the "Fed put". Speech at the Cato Institute, Washington, D.C., 30 November 2007.

Chapter 2

The role of liquidity constraints in the response of monetary policy to house prices

2.1 Introduction

Empirically there is a strong wealth effect on consumption spending. Conventional wisdom is that the marginal propensity to consume out of total net wealth is 3-5 cents per dollar (Altissimo et al., 2005). Furthermore, various studies find a stronger wealth effect of housing than of stock wealth for the U.S. (e.g. Davis and Palumbo, 2001; Case, Quigley and Shiller, 2001; Carroll, Otsuka and Slacalek, 2006). The difference may be explained by the more even distribution of housing wealth than of stock wealth across households, with a owner-occupier rate of nearly 70% in the U.S. and housing representing a larger part of total household wealth than equities (Illing and Klüh, 2005).

From a theoretical perspective it is not straightforward to justify the wealth effect from housing[1]. Consider a representative infinitely lived agent who owns the house in which she lives. An exogenous rise in house prices at a constant interest rate just compensates for the higher present value of expected future imputed rents. In this case the change in net wealth is zero and shouldn't have an effect on consumption. Even if the agent moved to a cheaper place, if housing services in the future improved, if higher collateral value resulted in saving on interest payments, or if the agent owned a high-value house but lived in a cheap one, there needn't be a wealth effect. Since the agent lives forever any change in net wealth is spread out into the infinite future and shouldn't affect consumption today. However, if the agent is liquidity constrained an increase in the value of the house can serve as additional collateral to borrow against. Housing value

[1] Carroll (2004) provides a discussion of this issue.

serves as a means to bring forward consumption and helps to smooth it over time, even though net worth hasn't changed[2]. In this case an increase in house prices can lead to an effect on consumption. Some authors argue theoretically and empirically that the process of financial liberalisation since the mid 1980s has increased the proportion of housing collateral that can be used to borrow against (e.g. Attanasio and Weber, 1994; Lustig and van Nieuwerburgh, 2006; Muellbauer et al., 1990; Ortalo-Magné and Rady, 2006). Others stress the role of rising house prices for a given level of financial liberalisation (e.g. Campbell and Cocco, 2007; Carroll, 2004). In the long run the fraction of liquidity constrained homeowners should decrease as financial innovation and liberalisation proceed and increase e.g. the loan-to-value ratio. In the short run the fraction of liquidity constrained agents varies because the possibility to smooth consumption depends on the level of house prices for given financial instruments. A sufficient increase in house prices is necessary for home-owners to benefit from the possibility of housing equity withdrawal. Housing equity withdrawal is the difference between net lending secured on housing and households' gross investment in housing (Bank of England). This way homeowners can increase their mortgage, i.e. cash flow, by a fraction of the increase in the value of their house[3]. Therefore it is clear that the fraction of liquidity constrained agents is not constant over time.

Housing equity withdrawal in the US and the UK has indeed increased considerably in the early 2000s at the same time as house prices increased as documented in figures (2.1) and (2.2). The simple correlation coefficient of the two series for the US is 0.83, while the one for the UK is smaller at 0.35.

Furthermore, Hurst and Stafford (2004) have shown that households do indeed use housing equity to smooth consumption in the face of an adverse shock such as unemployment[4]. For an economically significant effect on consumption a sufficiently large fraction of households must be homeowners and liquidity constrained. Figures (2.3) and (2.4) show the distribution of liquid asset and income, respectively, across U.S. homeowners in 2003. Clearly, a non-negligible share of homeowners have liquid assets of at most \$1000 and earn at most \$30000 per year[5].

The objective of this paper is to derive the implications of time-varying liquidity constraints for the optimal conduct of monetary policy. In the long

[2] A wealth effect of housing could also arise with finitely lived agents who don't care about the utility of their descendents. However, the focus here is one the role of liquidity constraints.

[3] For the construction of housing equity withdrawal from the data, see Greenspan and Kennedy (2005, 2007).

[4] Another use of housing equity would be to reoptimise the financial portfolio and not to spend it on consumption.

[5] Of course, what also matters is the history of assets and income. The percentage of homeowners with liquid assets of at most \$1000 and an annual income of at most \$30000 is 0.12 in the sample. For cut-off values of \$6200 for liquid assets and \$58800 for income as chosen by Hurst and Stafford (2004) the number is 0.35.

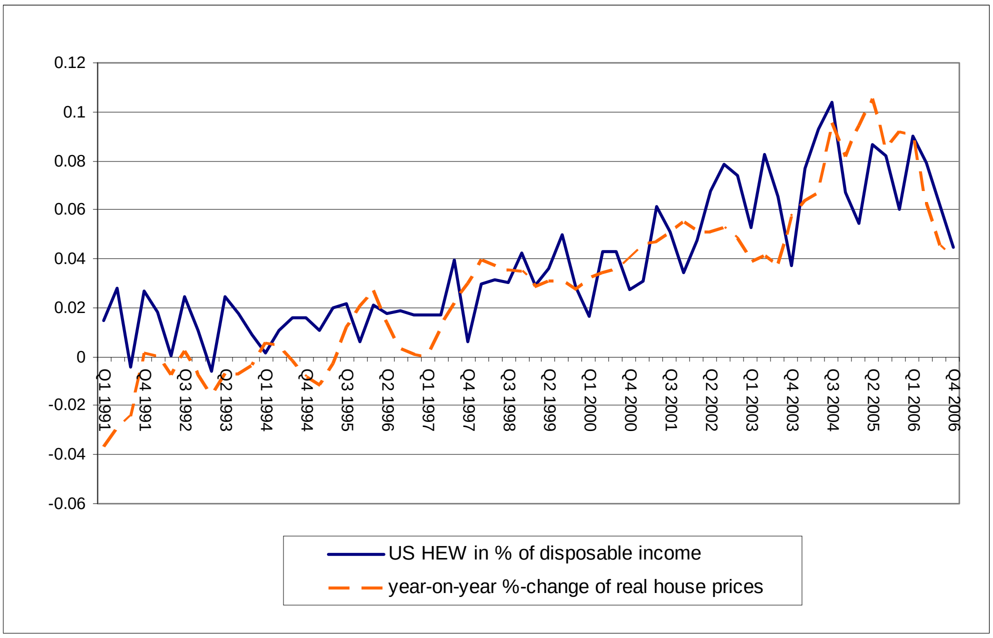

Figure 2.1: Housing equity withdrawal in % of disposable income (solid line) and the year-on-year real house price change (dashed line) in the US.
Notes: House prices deflated by the CPI. Source: Greenspan and Kennedy (2005, 2007).

run financial innovation should reduce the volatility of consumption and output through an increase in the fraction of consumption smoothers in the economy. However, in the short run house prices are volatile and affect the capacity of constrained households to borrow and thereby smooth consumption. Rising house prices allow for higher equity withdrawal boosting consumption, while falling house prices may make debtor households bankrupt or at least liquidity constrained depressing consumption. The contribution of this paper is to take account of the fact that higher house prices temporarily reduce the fraction of constrained households, who become consumption smoothers, while falling house prices temporarily increase it. The question asked is how monetary policy should react to house prices and the corresponding time-varying liquidity constraints.

A wealth effect from housing is derived by assuming that young home-owners are liquidity constrained in the sense that they have high permanent income relative to current income as it is typical for the life-cycle pattern of income. To the extent that they are owner-occupiers a rise in house prices enables them to extract the extra value and increase their consumption towards the optimal level as implied by the permanent income hypothesis. This way house prices increase aggregate demand and affect the output gap and inflation.

Our main results are that monetary policy should react to house price movements due to their effect on consumption by constrained agents. Moreover, with time-varying liquidity constraints, the optimal weights on expected inflation, the

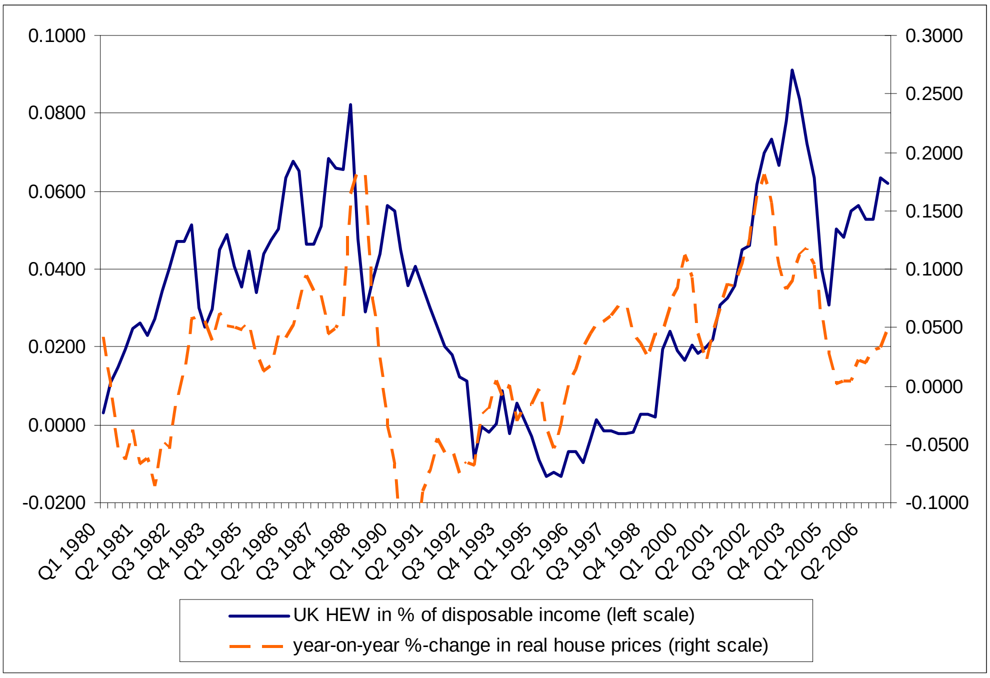

Figure 2.2: Housing equity withdrawal in % of disposable income (solid line) and the year-on-year real house price change (dashed line) in the UK.
Notes: House prices deflated by the CPI. Source: Datastream, own calculations.

output gap and house price changes are affected. It is one of the main contributions of the chapter to work out explicitly this mechanism. To the best of our knowledge this has not been looked at yet. Our results are of interest because they show that it is not only the house prices per se that matter but also their interaction with liquidity constraints and the associated effect on the weight on expected inflation and output in the optimal interest rate rule. This gives additional information to the policy maker about the strength of the optimal interest rate response to house prices. The optimal interest rate response crucially depends on the sensitivities of a change in the share of constrained agents with respect to house prices, expected inflation, the output gap and the interest rate.

The chapter is structured as follows. Section 2 relates the chapter to the literature. Section 3 sets up a life-cycle model of consumption and derives an IS curve with liquidity constraints. Section 4 derives the optimal monetary policy in a New Keynesian framework and a wealth effect from housing. Section 5 analyses the optimal interest rate response when there are time-varying liquidity constraints. Section 6 discusses some robustness checks of the model and section 7 concludes.

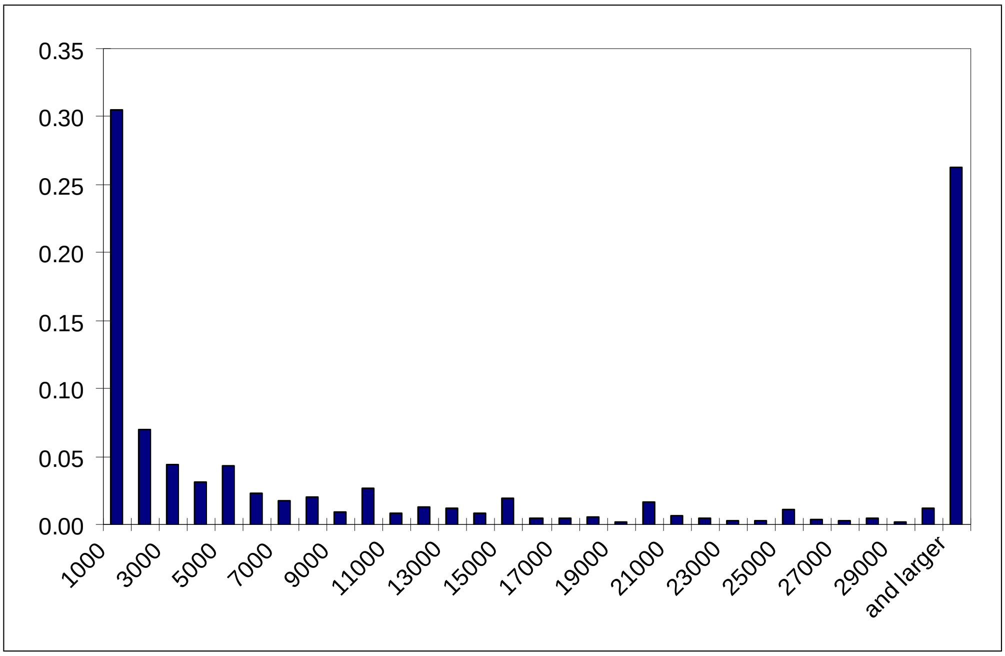

Figure 2.3: Distribution of liquid assets in 2003 $ across U.S. homeowners. Notes: Liquid assets are the sum of stocks, checking and savings accounts, money market funds, certificate of deposits, government savings bonds, treasury bills, bond funds and life insurances. Source: PSID, own calculations.

2.2 Related literature

The present chapter relates to a vast amount of papers analysing the relationship of monetary policy and asset prices. Typically they don't distinguish between different types of assets. Broadly speaking there are two main questions in the context of the optimal response of monetary policy to asset prices. The first is how should monetary policy react to asset prices over and above a conventional wealth effect from asset prices, especially bubbles. Two approaches can be found in the literature. One looks at demand effects from asset prices (Bernanke and Gertler, 1999, 2001; Cecchetti et al., 2000; Greenspan, 1999, 2004; Gruen, Plumb and Stone, 2005; Filardo, 2004; Kent and Lowe, 1997; Kontonikas and Montagnoli, 2006). In this approach a developing and consequently bursting bubble might lead to household and firm bankruptcies, thereby affecting the output gap and inflation. A sufficiently forward looking central bank might want to take these repercussions into account. This argument suggests adjusting the central bank's forecast horizon for expected inflation to include periods of possible asset bubble bursts. There is disagreement, however, about how to identify a bubble with certainty, about the timing, direction and strength of the warranted interest rate response. Also a pre-emptive restrictive monetary policy at the expense of current output might be hard to justify to the public.

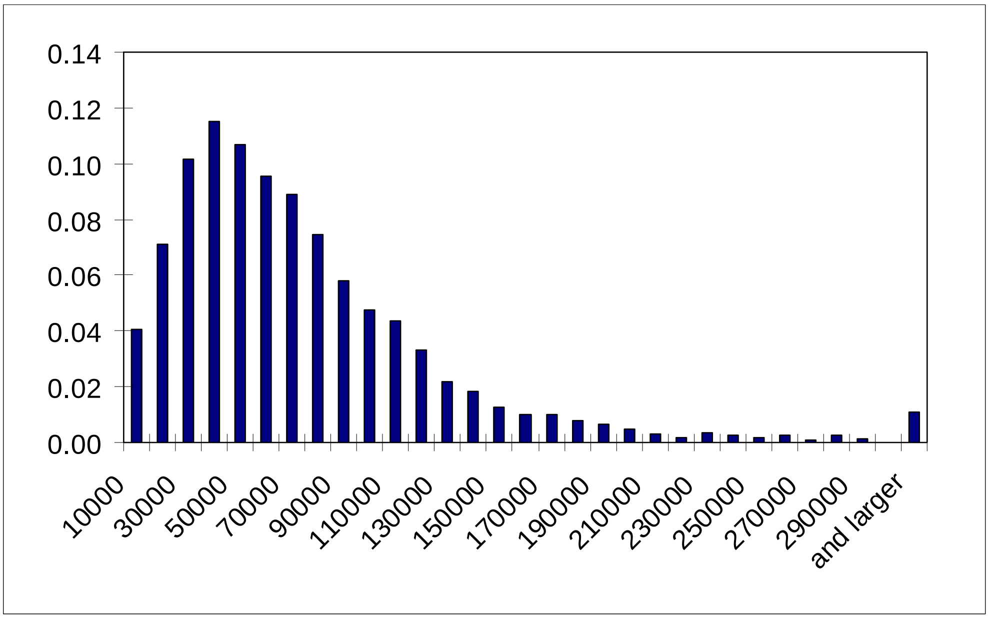

Figure 2.4: Distribution of annual income in 2002 $ across U.S. homeowners. Notes: Annual income is reported income in 2003 about tax year 2002. Source: PSID, own calculations.

Another approach looks at the supply effects of asset prices[6]. Bean (2004) sets up a model drawing on results from a study by Borio and Lowe (2002) where asset prices are correlated with the build-up of debt, which is used to finance capital accumulation. An asset bubble crash leads to a credit crunch, which affects total factor productivity due to the lack of funds from intermediaries. The output gap suddenly widens with adverse effects on inflation. One way in which monetary policy can affect the probability of a credit crunch is to deter the debt build-up. In the model this can be achieved by a policy under committment where the central bank affects expectations of future output gaps. A higher interest rate leads to a lower expected future output gap, which in turn means slower capital accumulation today. Correspondingly, this limits the build-up of debt. Thus, an interest rate response over and above the one warranted by expected inflation and the current output gap is optimal. Bordo and Jeanne (2002) argue in a similar way that raising the interest rate today to bring down debt accumulation can be considered an insurance against negative future supply shocks when asset prices crash. In their model the real interest rate directly affects firm's demand for debt.

The second question is about the mechanism of the wealth effect, by which asset prices (stock or house prices) affect consumption and the appropriate policy reaction. When looking at the channel from asset prices to consumption it is

[6]At the intersection of demand and supply effects is a paper by Smets (1997), who focuses on the informational content in asset prices for expected inflation.

important to distinguish different classes of assets. House and stock prices can have different effects on consumption, e.g. stock-ownership is much less widely spread than home-ownership in the U.S. Then again house price increases do not necessarily always represent increases in net wealth. Yet many papers commonly just append a variable for asset prices to the IS equation or directly to the interest rate rule. In contrast in this paper we focus on the role of house prices and explicitly derive a wealth effect from liquidity constrained consumers. We can show that the precise channel by which house prices affect consumption is important because the weights on inflation, output and house prices in the interest rate rule are affected. Our paper relates most closely to the papers by Iacoviello (2004, 2005) and Monacelli (2006) who also derive a wealth effect from asset prices from a microfounded model. Some home-owners are assumed to be impatient while others are patient. This determines who becomes borrower or lender. In Iacoviello (2005) borrowing capacity is limited by the expected future value of the house such that a house price increase results in higher consumption by borrowers. He analyses optimal monetary policy using a postulated interest rate rule, instead of deriving it from a loss function. In Monacelli (2006) borrowers are constrained by the value of their general assets. He analyses to which extent it might be optimal for a central bank, which maximises the weighted utility of borrowers and savers, to deviate from price stability when inflation erodes the real value of debt and relaxes borrowing constraints. In our paper liquidity constrained consumers are essentially defined by age and the value of their home, which is intuitive and corresponds well with the life-cycle pattern of income. It allows to let the share of constrained agents vary over time. In contrast, when constraints are defined by a fixed rate of time preference this is not possible. Moreover, we explicitly exclude the possibility of precautionary saving to be able to uniquely determine when liquidity constraints are binding and when not. Furthermore, we derive an interest rate rule from loss minimisation by the central bank.

Time-varying liquidity constraints have been considered e.g. by Deaton (1991), Ludvigson (1999) and Pesaran and Smith (1995). Commonly, constraints are a complex function of past income and net asset accumulation. This makes most models with time-varying liquidity constraints intractable. Therefore we aimed at finding a way to make liquidity constraints independent of past values of income and assets and only conditional on the actual value of the home, albeit at the expense of a more stylised setup.

To sum up, the contributions of our paper are first to derive an explicit wealth effect from house prices on consumption via relaxing liquidity constraints, and second to analyse optimal monetary policy when liquidity constraints vary over time with house prices.

2.3 A life-cycle model of consumption

Since the aim of the analysis is to evaluate monetary policy with time-varying liquidity constraints in a standard New Keynesian setup we first derive the IS curve from individual utility optimisation taking into account that a fraction of households is liquidity constrained and consumes out of current income and liquid assets. Together with a Phillips curve and the central bank's loss function we derive the optimal monetary policy under constant and under time-varying liquidity constraints.

Typically, the IS curve in the New Keynesian model is derived from household utility maximisation using a standard utility function such as the CES utility. In this model we use a quadratic utility function because we want to separate precautionary saving from liquidity constraints as a source for the high correlation of current income and liquid assets with current consumption for the constrained agents. The marginal utility of a quadratic utility function is linear, which implies that the expected marginal utility of consumption equals the marginal utility of expected consumption. An increase in uncertainty about future consumption doesn't affect marginal utility, i.e. certainty equivalence holds. Therefore there is no effect on current consumption and saving[7]. The precautionary saving motive may result in consumption that follows current income closely and is observationally similar to the effect of liquidity constraints (Carroll, 1997). An agent may save little and consumption might follow current income closely either because the agent is liquidity constrained, or because the agent is not liquidity constrained, and would want to borrow as much as necessary to attain a smooth consumption path, but the percautionary saving motive counteracts the desire to borrow just so that consumption and current income are closely correlated.

Moreover, to have borrowing in equilibrium some agents must be constrained and others not. Therefore we build a model with three types of agents: young, middle-aged and old. In every period all types coexist and all are owner-occupiers of their house.

The main challenge of the model is to avoid having to account for the history of assets and income in determining when an agent is constrained. To this end it is assumed that all agents face the same hump-shaped profile of life-time income and only the young agents can be constrained. In each period the three type of agents differ in the shares of aggregate income they receive as well as in their share of total consumption. Thus each agent's income is a fixed share of aggregate income and so is her consumption.

[7]With quadratic utility, however, possibly binding liquidity constraints in the future may affect current consumption (see Romer's textbook, 2001). This will be ruled out by assumption.

2.3.1 Unconstrained consumers

Without any borrowing constraints a young agent in period t maximises her life-time utility subject to her life-time budget constraint.

$$\max_{\{C_{1t},C_{2t+1},C_{3t+2}\}}$$
$$\begin{aligned} U\left(C_{1t},C_{2t+1},C_{3t+2}\right) = \left(C_{1t}-aC_{1t}^2\right)+\beta E_t\left(C_{2t+1}-aC_{2t+1}^2\right) \\ +\beta^2 E_t\left(C_{3t+2}-aC_{3t+2}^2\right) \end{aligned}$$

$$\text{s.t.}$$
$$\begin{aligned} C_{1t}+\frac{1+\pi_{t+1}}{1+i_t}C_{2t+1}+\frac{1+\pi_{t+1}}{1+i_t}\frac{1+\pi_{t+2}}{1+i_{t+1}}C_{3t+2} = \\ Y_{1t}+\frac{1+\pi_{t+1}}{1+i_t}Y_{2t+1}+\frac{1+\pi_{t+1}}{1+i_t}\frac{1+\pi_{t+2}}{1+i_{t+1}}Y_{3t+2} \end{aligned}$$

where $a > 0$, C_{jt} and Y_{jt} are consumption and income, respectively, of agent j in period t, and $j = \{1,2,3\}$ denotes young, middle-aged and old agents. a determines the curvature of the utility function and β is the discount factor. i_t is the nominal interest rate during period t and π_{t+1} is the inflation rate from period t to $t+1$. The first-order conditions with respect to C_{1t}, C_{2t+1}, C_{3t+2} are

$$\begin{aligned} 1-2aC_{1t}-\mu &= 0 \\ \beta E_t\left(1-2aC_{2t+1}\right)-E_t\frac{1+\pi_{t+1}}{1+i_t}\mu &= 0 \\ \beta^2 E_t\left(1-2aC_{3t+2}\right)-E_t\frac{1+\pi_{t+1}}{1+i_t}\frac{1+\pi_{t+2}}{1+i_{t+1}}\mu &= 0 \end{aligned}$$

with μ being the Lagrange multiplier. They can be written more compactly in form of two Euler equations for the two adjacent periods:

$$C_{1t} = -\beta E_t\left(\frac{1+i_t}{1+\pi_{t+1}}\right)\left(\frac{1}{2a}-C_{2t+1}\right)+\frac{1}{2a} \tag{2.1}$$
$$C_{2t+1} = -\beta E_{t+1}\left(\frac{1+i_{t+1}}{1+\pi_{t+2}}\right)\left(\frac{1}{2a}-C_{3t+2}\right)+\frac{1}{2a} \tag{2.2}$$

Note that in the special case of a constant real interest rate of zero and a discount factor of one optimal consumption is equal across the three periods. In the general case, log-linearisation of (2.1) and (2.2) results in the following consumption equations

$$c_{1t} = \phi_1 E_t c_{2t+1}-\phi_2\left(i_t-E_t\pi_{t+1}\right) \tag{2.3}$$
$$c_{2t+1} = \phi_3 E_{t+1}c_{3t+2}-\phi_4\left(i_{t+1}-E_{t+1}\pi_{t+2}\right) \tag{2.4}$$

From here on lower case letters denote percentage deviations from trend. ϕ_1, ϕ_2, ϕ_3, ϕ_4 are positive linearisation constants[8]. Given the finite lives of agents one needs to specify what happens to housing wealth at the end of the third period of an agent's life. If there were no bequests a house price rise would have an effect on consumption of the old. Since they have only one last period to live they would consume all their remaining housing wealth. Since the focus of the paper is on the role of liquidity constraints as a housing wealth channel, we shut down the wealth effect from finite lives by implicitly assuming that agents care for their descendants and bequeath their total remaining housing wealth at the end of the third period to the middle-aged agents. This way housing wealth always either exactly compensates for future imputed rents or is spread into the infinite future such that the net change in housing wealth is always zero.

2.3.2 Who is constrained and why?

To work out the role of liquidity constraints in the transmission mechanism from house prices to consumption as simply as possible, it is assumed that only the young agents can be constrained. Japelli (1990) reports not having a credit history or the age of the loan applicant as the single most frequent reason given by lenders when they rejected loan applications. Constrained young agents just consume their current income plus liquid assets.

$$c^c_{1t} = \psi_1 y_{1t} + \psi_2 b_t$$

where ψ_1, ψ_2 are positive linearisation constants[9]. Suppose that liquid assets consist only of housing equity withdrawal, which in turn depends on the house price change q_t

$$b_t = bq_t$$

where b measures the extent to which an increase in house prices can be cashed in.

In the model the young are constrained if desired consumption according to utility optimisation and consumption smoothing c^u_{1t} is larger than current income y_{1t} and liquid assets b_t, "cash-on-hand".

$$c^u_{1t} > \psi_1 y_{1t} + \psi_2 bq_t \tag{2.5}$$

It is in addition assumed that the middle-aged and the old are always uncon-

[8] $\phi_1 = \frac{\beta\left(\frac{1+i_0}{1+\pi_0}\right)C_{20}}{C_{10}}$, $\phi_2 = \frac{\beta\left(\frac{1}{2a}-C_{20}\right)\frac{1+i_0}{1+\pi_0}}{C_{10}}$, $\phi_3 = \frac{\beta\left(\frac{1+i_0}{1+\pi_0}\right)C_{30}}{C_{20}}$, $\phi_4 = \frac{\beta\left(\frac{1}{2a}-C_{20}\right)\frac{1+i_0}{1+\pi_0}}{C_{20}}$, $U'_{C_{j,t+j-1}} = 1 - 2aC_{j,t+j-1} > 0$

[9] $\psi_1 = \frac{Y_{10}}{C_{10}} < 1, \psi_2 = \frac{B_0}{C_{10}} < 1$

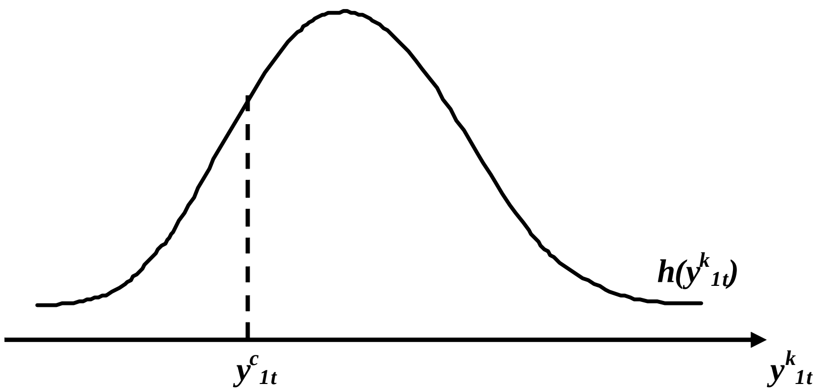

Figure 2.5: Distribution function of income going to the young agents

strained. Typically, life-time income is hump-shaped (Attanasio and Browning, 1995; Campbell and Cocco, 2007; Carroll, 1997; Gourinchas and Parker, 2002) and consumption smoothing implies borrowing from the middle-aged when young and paying off the debt to the old in the following period[10]. However, future income of young agents is not pledgeable, unless they use the value of their house as collateral.

2.3.3 Time-varying liquidity constraints

As explained in the introduction, the capacity of homeowners to withdraw equity from their houses varies over time as house prices vary. Therefore the share of constrained agents in the economy should vary too. Typically in existing models of monetary policy and house prices this aspect is not taken into account and the share of constrained agents is fixed (e.g. Iacoviello, 2005). We relax this assumption by making the share of constrained agents a function of the house price. While the total amount of income going to the young is fixed, we assume that the income going to an individual young agent k, denoted by y_{1t}^{k}, is distributed over all young agents according to some distribution function $h\left(y_{1t}^{k}\right)$, which is illustrated in figure 2.5.

For young agents with income below y_{1t}^{c} and a given amount of housing equity withdrawal liquid assets are insufficient to cover desired consumption and they are constrained. For young agents with income above y_{1t}^{c} and a given amount of housing equity withdrawal liquid asset are enough to cover desired consumption and they are unconstrained. y_{1t}^{c} is the income of a young agent that just makes her unconstrained, since her liquid assets just cover her desired optimal consumption, $c_{1t}^{u} = \psi_1 y_{1t}^{c} + \psi_2 b q_t$, which can be rearranged and substituted in

[10] The ability to borrow in equilibrium is the reason to have three types of agents.

to

$$y_{1t}^{c} = \frac{\phi_1}{\psi_1} E_t c_{2t+1} - \frac{\phi_2}{\psi_1} \left(i_t - E_t \pi_{t+1} \right) - \frac{\psi_2 b}{\psi_1} q_t$$

The proportion of constrained agents is the share of young agents with income below that critical level.

$$\alpha_t = \int_0^{y_{1t}^{c}} h\left(y_{1t}^{k}\right) dy_{1t}^{k} = F\left(\frac{\phi_1}{\psi_1} E_t c_{2t+1} - \frac{\phi_2}{\psi_1} \left(i_t - E_t \pi_{t+1} \right) - \frac{\psi_2 b}{\psi_1} q_t \right)$$

The share of constrained agents depends on expected future consumption, the real interest rate and real house prices and not the entire history of income and assets. This is by construction to keep the model tractable. When expected future consumption rises, more agents are constrained ceteris paribus since optimal desired consumption rises. Similarly, when the nominal interest rate falls or expected inflation rises, the real interest rate falls ceteris paribus and optimal desired consumption increases making more agents constrained. Finally, note that the proportion of constrained agents falls ceteris paribus with higher house prices. This is because higher house prices allow to withdraw equity from the house, which can be used to finance consumption.

Note that the house price q_t possibly also depends on the interest rate i_t.

$$q_t = q_t\left(i_t\right)$$

There are, however, arguments for why monetary policy should not expect to be able to influence asset prices via interest rate changes in a boom phase. Even with higher interest rates expectations might be sufficiently optimistic to overcompensate the dampening effect of higher interest rates. As a start we will take house prices to be exogenous, while later on relaxing that assumption.

2.3.4 Aggregation and equilibrium

Aggregate consumption c_t is the sum of the weighted consumption of the young, the consumption of the middle-aged and the old.

$$c_t = \left(1 - \alpha_t\right) c_{1t}^{u} + \alpha_t c_{1t}^{c} + c_{2t} + c_{3t}$$

As in the standard model of aggregate consumption we use the Euler equations (2.3) and (2.4) to determine each agent's consumption at t and aggregate, which yields

$$\begin{aligned}
c_t &= (1-\alpha_t)\left[\phi_1 E_t c_{2t+1} - \phi_2\left(i_t - E_t\pi_{t+1}\right)\right] + \alpha_t\left[\psi_1 y_{1t} + \psi_2 b q_t\right] \\
&\quad + \phi_3 E_t c_{3t+1} - \phi_4\left(i_t - E_t\pi_{t+1}\right) \\
&\quad + \phi_5 E_t c_{2t+1} - \phi_6\left(i_t - E_t\pi_{t+1}\right)
\end{aligned}$$

The first line is the weighted average of constrained and unconstrained young agents, the second line the consumption of the middle-aged and the third line is the consumption of the old. Consumption of the old is a usual Euler equation under the assumption that the old care about consumption of their descendants, who are middle-aged in the following period. This assumption is innocuous with regard to the qualitative results of the model and follows the assumption above about housing bequests[11]. It is justified by the focus of the paper on housing as a means to bring forward consumption in time, as opposed to a wealth effect from housing from finite lives.

In equilibrium $c_t = y_t$ must hold. In addition, as stated above, each agent faces the same life-time pattern of income and receives a fixed fraction s_j of aggregate income. In particular the income of the young $y_{1t} = s_1 y_t$. This assumption does not mean that income is predetermined. Rather as in the standard New Keynesian model it is demand determined. Also note that while the share of income going to the young is a fixed fraction of aggregate income, it is is distributed over the young agents as specified above. Moreover, each agent consumes a fixed fraction x_j of aggregate income[12].

$$\begin{aligned}
c_{1t} &= x_1 y_t \\
c_{2t} &= x_2 y_t \\
c_{3t} &= x_3 y_t
\end{aligned}$$

Using these assumptions results in the following IS curve.

$$\begin{aligned}
y_t &= (1-\alpha_t)\left[\phi_1 x_2 E_t y_{t+1} - \phi_2\left(i_t - E_t\pi_{t+1}\right)\right] + \alpha_t\left[\psi_1 s_1 y_t + \psi_2 b q_t\right] \\
&\quad + \phi_3 x_3 E_t y_{t+1} - \phi_4\left(i_t - E_t\pi_{t+1}\right) \\
&\quad + \phi_5 x_2 E_t y_{t+1} - \phi_6\left(i_t - E_t\pi_{t+1}\right) \qquad (2.6)
\end{aligned}$$

As usual the IS curve is increasing in the expected future output gap, decreasing in the real interest rate. In addition and in contrast to the representative

[11] Consumption of the old could alternatively be set to their permanent income. This would, however, also involve past values of income, the nominal interest rate and inflation rate, as well as the current inflation rate. This would make no qualitative difference, while decreasing tractability of the model.

[12] Note that as long as $x_1 \neq x_2 \neq x_3$, $c_{10} \neq c_{20} \neq c_{30}$.

infinitely lived agent, there is an explicit wealth effect from housing through housing equity withdrawal by the constrained young agent. Moreover, aggregate consumption and income depend on the share of constrained agents, on their current income and on current inflation due to consumption of the old. The share of constrained agents is now

$$\alpha_t = F\left(\frac{\phi_1 x_2}{\psi_1} E_t y_{t+1} - \frac{\phi_2}{\psi_1}\left(i_t - E_t \pi_{t+1}\right) - \frac{b\psi_2}{\psi_1} q_t\right) \tag{2.7}$$

2.4 Optimal monetary policy

In the long-run financial liberalisation such as the introduction of housing equity withdrawal or gradually rising loan-to-value ratios (see Ortalo-Magné and Rady, 1999) alleviate borrowing constraints on consumers if they have permanently better access to credit. This could in principle help consumers to better smooth consumption and therefore make output and inflation less variable. Monetary policy makers would welcome it provided financial liberalisation doesn't increase financial instability. In this paper, however, we are concerned with the short-run implications of financial liberalisation. In particular, how should monetary policy react to house price movements when, combined with financial innovations such as housing equity withdrawal, these result in variation of the share of liquidity constrained consumers in the economy? When house prices rise constrained consumers are able to expand their consumption, which leads to a wealth effect from house prices in the model above. However, at the same time some previously constrained consumers become unconstrained, which reduces the share of constrained agents in the economy. From (2.6) it is then not immediately clear anymore how the output gap is affected and how monetary policy should respond.

To analyse optimal monetary policy we use the standard New Keynesian framework as e.g. in Walsh (2003). The key innovation in the paper is, however, the modified IS curve (2.6), which is reproduced here for convenience.

$$\begin{aligned}
y_t \quad = \quad & (1-\alpha_t)\left[\phi_1 x_2 E_t y_{t+1} - \phi_2\left(i_t - E_t\pi_{t+1}\right)\right] + \alpha_t\left[\psi_1 s_1 y_t + \psi_2 b q_t\right] \\
& + \phi_3 x_3 E_t y_{t+1} - \phi_4\left(i_t - E_t\pi_{t+1}\right) \\
& + \phi_5 x_2 E_t y_{t+1} - \phi_6\left(i_t - E_t\pi_{t+1}\right)
\end{aligned}$$

Furthermore, there is a forward looking Phillipscurve

$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t + e_t \tag{2.8}$$

where π_t is inflation from period $t-1$ to t, β is the discount factor, E_t the expectations operator as of period t, κ is the impact of the output gap on inflation and e_t is a cost push shock, which obeys

$$e_t = \rho e_{t-1} + \widehat{e}_t \tag{2.9}$$

with $0 < \rho \leq 1$, and $\hat{e}_t$ is an i.i.d. random variable with zero mean and constant finite variance. Finally, the central bank's loss function is specified as

$$L_t = \frac{1}{2} E_t \sum_{i=0}^{\infty} \beta^i \left(\pi_{t+i}^2 + \lambda y_{t+i}^2 \right)$$

where λ is the weight the central bank puts on deviations of the output gap from target. Moreover, since the focus of the paper is on house prices and time-varying liquidity constraints we keep it as simple as possible and derive the optimal policy under discretion. To eliminate an inflation bias under discretion we assume a target for the output gap of zero. The monetary policy maker minimises in every period the loss function L_t subject to the Phillipscurve using the Lagrangean Λ_t.

$$\Lambda_t = \frac{1}{2} E_t \sum_{i=0}^{\infty} \beta^i \left(\pi_{t+i}^2 + \lambda y_{t+i}^2 \right) + \theta_t \left(\pi_t - \beta E_t \pi_{t+1} - \kappa y_t - e_t \right)$$

where θ_t is the Lagrange multiplier on the Phillipscurve. The IS curve is no constraint on monetary policy as long as it can costlessly vary the nominal interest rate[13]. The first order conditions for optimal monetary policy are

$$\begin{aligned} \text{w.r.t. } \pi_t \quad &: \quad 2\pi_t + \theta_t = 0 \\ \text{w.r.t. } y_t \quad &: \quad 2\lambda y_t - \theta_t \kappa = 0 \end{aligned}$$

which can be written more compactly as

$$\pi_t = -\frac{\lambda}{\kappa} y_t \tag{2.10}$$

The optimality condition states that the marginal cost in terms of higher inflation must be equal to the marginal benefit of a larger output gap. The central bank trades off inflation against the output gap taking into account its preferences and the Phillipscurve. Using the optimality condition (2.10), the AR(1) process of the cost push shock (2.9) and the Phillipscurve (2.8) in the IS curve (2.6) yields the interest rate rule as a function of the optimal inflation rate and

[13] Formally including the IS curve in the optimisation problem leads to a Lagrange multiplier of zero for the IS curve constraint. Modifications of the setup that only affect the IS curve don't change the first order conditions of the standard setup under discretion.

output gap as well as house prices[14]

$$i_t = f_\pi E_t \pi_{t+1} + f_y y_t + f_q q_t \tag{2.11}$$

$$\begin{aligned}
f_\pi &= 1 + \frac{(1-\alpha_t \psi_1 s_1)\,\kappa}{\rho\lambda\left(\phi_6 + \phi_4 + \phi_2\left(1-\alpha_t\right)\right)} > 1 \\
f_y &= \frac{\left(\left(1-\alpha_t\right)\phi_1 x_2 + \phi_3 x_3 + \phi_5 x_2\right)\rho}{\phi_6 + \phi_4 + \phi_2\left(1-\alpha_t\right)} > 0 \\
f_q &= \frac{b\psi_2 \alpha_t}{\phi_6 + \phi_4 + \phi_2\left(1-\alpha_t\right)} > 0
\end{aligned}$$

The coefficient on expected inflation is positive and larger than 1, the coefficient on the output gap is positive and the coefficient on the house price is positive too. Moreover, using the optimality condition (2.10) in the definition of the share of constrained agents results in

$$\alpha_t = F\left(\frac{\phi_1 x_2}{\psi_1}\rho y_t - \frac{\phi_2}{\psi_1} i_t + \frac{\phi_2}{\psi_1} E_t \pi_{t+1} - \frac{\psi_2 b}{\psi_1} q_t\right) \tag{2.12}$$

Note that the cumulative distribution function F has the following characteristics.

$$\begin{aligned}
\frac{\partial F}{\partial i_t} &\equiv \alpha_i' < 0 \\
\frac{\partial F}{\partial\left(E_t \pi_{t+1}\right)} &\equiv \alpha_\pi' > 0 \\
\frac{\partial F}{\partial y_t} &\equiv \alpha_y' > 0 \\
\frac{\partial F}{\partial q_t} &\equiv \alpha_q' < 0
\end{aligned}$$

2.5 The role of liquidity constraints

Having derived an interest rate rule for monetary policy in (2.11) we are now able to analyse the role of house prices and the associated time-varying liquidity constraints in the conduct of monetary policy. From (2.11) it is clear that the optimal policy implies an interest rate response to expected inflation, output and to house prices. Moreover, however, the weights on each variable depend on the share of constrained agents α_t, which in turn varies with y_t, i_t, $E_t\pi_{t+1}$ and q_t.

[14]Details in the appendix.

2.5.1 Constant liquidity constraints

Consider, as a benchmark, the simple case in which liquidity constraints are constant, $\alpha_t = \alpha$. Monetary policy should react to the house price shock with a weight given by

$$\left.\frac{di_t}{dq_t}\right|_{\alpha \text{ const.}} = f_q = \frac{b\psi_2\alpha}{\phi_6 + \phi_4 + \phi_2(1-\alpha)} > 0 \qquad (2.13)$$

Monetary policy should thus respond to rising house prices by increasing the interest rate. It has been shown that optimal monetary policy in the New Keynesian model should respond to a wealth effect from asset prices only to the extent that they affect the output gap and inflation expectations (Bean, 2004). This means the policy-maker needn't worry about asset prices themselves if they have only little information about their movements. Instead it is enough to observe the output gap and respond accordingly[15]. The same result holds here when liquidity constraints are constant. The extent of an interest rate reaction to house price movements depends on the degree to which liquidity constrained consumers are able to convert the increased value of their home into cash and ultimately into consumption, as denoted by the parameter b. Furthermore, the optimal weights on the output gap and expected inflation are given by (2.11) with $\alpha_t = \alpha$. While an increase in house prices or other factors affecting consumption, e.g. an increase in expected future income and consumption, could in principle be judged only by their impact on the output gap the separation of the two effects in this model allows to get information about the strength of the appropriate response since the coefficients on aggregate consumption, i.e. in equilibrium the output gap, and house prices differ.

For illustration, let's look at the extreme case where monetary policy has to deal either with the young agents all constrained or all unconstrained. If α was equal to 1 the optimal rule suggests reacting to the house price shock with a weight

$$f_q|_{\alpha=1} = \frac{b\psi_2}{\phi_6 + \phi_4} > 0$$

The weights on expected inflation and the output gap are then given by

$$\begin{aligned} f_\pi|_{\alpha=1} &= 1 + \frac{(1-\psi_1 s_1)\kappa}{\rho\lambda(\phi_6+\phi_4)} > 1 \\ f_y|_{\alpha=1} &= \frac{(\phi_3 x_3 + \phi_5 x_2)\rho}{\phi_6+\phi_4} > 0 \end{aligned}$$

If α turned out to be 0 we are back in the standard scenario without a wealth

[15] If asset prices conveyed better infomation about the underlying state of the economy than the output gap, it might theoretically be better to respond to them and not the output gap.

effect from house prices. Consequently there is no separate response to house prices required over and above the one to expected inflation and the output gap since there are only unconstrained agents, whose consumption doesn't react to house prices. The weights on expected inflation and the output gap are

$$\begin{aligned} f_\pi|_{\alpha=0} &= 1 + \frac{\kappa}{\rho\lambda\left(\phi_6 + \phi_4 + \phi_2\right)} > 1 \\ f_y|_{\alpha=0} &= \frac{\left(\phi_1 x_2 + \phi_3 x_3 + \phi_5 x_2\right)\rho}{\phi_6 + \phi_4 + \phi_2} > 0 \end{aligned}$$

The model shows that the weights on expected inflation and the output gap differ in the two cases. Whether the response to expected inflation is smaller when all young agents are constrained, $\alpha = 1$, compared to all agents being unconstrained, $\alpha = 0$, depends on the share of income s_1 going to the constrained. The fact that the constrained don't react to the interest rate would by itself call for a stronger interest rate response to bring down inflation by a given amount. However, to bring down inflation requires a fall in the outgap, i.e. income. Since the constrained consume out of current income, their consumption falls and with it the pressure on the output gap. This compensates for a stronger interest rate response. In particular, the weight on expected inflation is smaller when all young agents are constrained if the share of income going to the young s_1 is large enough, as defined by $s_1 > \frac{\phi_2}{\psi_1(\phi_2+\phi_4+\phi_6)}$.

The weight on the output gap f_y is larger when $\alpha = 1$ than when $\alpha = 0$ if $x_2 < \frac{\phi_2\phi_3 x_3}{\phi_1(\phi_6+\phi_4)-\phi_2\phi_5}$[16]. Key to understanding the effect here is that a given shock spreads to future expected output via the autocorrelated cost-push shock. An increase in the expected output gap increases the share of constrained agents because optimal consumption increases (see (2.12)). On the one hand the constrained don't respond to interest rate changes, which requires a stronger interest rate response. On the other hand they don't respond to future expected output anymore, so that pressure on the output gap is partly relieved. The pressure relieved is small if the share of consumption in income of the middle-aged is small. Then the first effect dominates and the weight on the output gap increases.

In addition, as will be discussed in the next section, if liquidity constraints are time-varying, house price movements have an impact on the weights in the optimal interest rate rule.

2.5.2 Time-varying liquidity constraints

Let now the share of liquidity constrained consumers be determined by (2.12). While the rule still suggests increasing the interest rate in the face of an increase in expected inflation, the ouput gap or house prices, the weights on these vari-

[16] Since $x_2 \geqq 0$, it must hold that $\phi_1\left(\phi_4 + \phi_6\right) > \phi_2\phi_5$.

ables now vary with the share of constrained agents. The share of constrained agents is positively related to the output gap and expected inflation and negatively related to the nominal interest rate and house prices. Consequently, each weight is a function of expected inflation, the output gap, the interest rate and house prices. We will discuss each weight in turn.

2.5.2.1 The optimal weight on expected inflation

When liquidity constraints are time-varying the optimal rule (2.11) not only suggests responding to house prices, but also that the optimal weights on the arguments in the rule change with the house price shock. Consider how the optimal weight on expected inflation changes with house price movements

$$\frac{df_{\pi}}{dq_t} = \frac{\alpha'_q \kappa \rho \lambda \left(\phi_2 - \psi_1 s_1 \left(\phi_2 + \phi_4 + \phi_6\right)\right)}{\left[\rho \lambda \left(\phi_6 + \phi_4 + \phi_2 \left(1 - \alpha_t\right)\right)\right]^2}$$

Since $\alpha'_q < 0$ the weight on expected inflation decreases with house prices if the share of income going to the young s_1 is small, $s_1 < \frac{\phi_2}{\psi_1(\phi_2+\phi_4+\phi_6)}$. Higher house prices decrease the share of constrained agents because they allow to extract equity from the house to finance consumption. Intuitively, the same two effects as above with constant α are at work. On the one hand, since an agent doesn't react to changes in the interest rate when constrained, but does so when unconstrained, a weaker interest rate response is required with more agents unconstrained to bring down expected inflation by a given amount. On the other hand, the constrained consume out of current income and liquid assets, while the unconstrained don't. An interest rate increase depresses current income and lowers consumption by the constrained, which helps to bring down inflation through an indirect channel. When this channel is partially shut down because more agents are unconstrained, it must be compensated by a stronger direct interest rate channel. However, the smaller the share of income s_1 going to the constrained, the weaker is the indirect effect that must be compensated. If $s_1 < \frac{\phi_2}{\psi_1(\phi_2+\phi_4+\phi_6)}$ the direct effect more than compensates the indirect effect and a weaker response to expected inflation is warranted.

Moreover, the weight on expected inflation is a function of expected inflation itself, via its effect on α_t.

$$\frac{df_{\pi}}{d\left(E_t \pi_{t+1}\right)} = \frac{\alpha'_{\pi} \kappa \rho \lambda \left(\phi_2 - \psi_1 s_1 \left(\phi_2 + \phi_4 + \phi_6\right)\right)}{\left[\rho \lambda \left(\phi_6 + \phi_4 + \phi_2 \left(1 - \alpha_t\right)\right)\right]^2}$$

which is positive if the share of income going to the young s_1 is small, $s_1 < \frac{\phi_2}{\psi_1(\phi_2+\phi_4+\phi_6)}$. To recapitulate the intuition, consider now an increase in the share of constrained agents. Higher expected inflation increases the share of constrained agents by raising the optimal level of consumption. Since the constrained don't react to changes in the interest rate, a stronger interest rate response is required. On the other hand, the newly constrained consume out of current income and liquid assets. An interest rate increase depresses their income and as such lowers consumption by the constrained. This effect however is smaller the smaller the share of income s_1 going to the constrained, calling for a stronger rate increase.

Similarly, the weight on expected inflation also increases with the output gap if $s_1 < \frac{\phi_2}{\psi_1(\phi_2+\phi_4+\phi_6)}$:

$$\frac{df_\pi}{dy_t} = \frac{\alpha_y' \kappa \rho \lambda \left(\phi_2 - \psi_1 s_1 \left(\phi_2 + \phi_4 + \phi_6\right)\right)}{\left[\rho \lambda \left(\phi_6 + \phi_4 + \phi_2 \left(1 - \alpha_t\right)\right)\right]^2}$$

Again with more agents constrained there are two effects: On the one hand a stronger interest rate increase is necessary to bring down expected inflation since fewer agents react to interest rate changes. On the other hand, the constrained react indirectly to the interest rate change as far as it affects aggregate income. If the indirect effect is small, because a small share of aggregate income goes to the constrained, then the first effect outweighs the second one and a stronger interest rate response is warranted.

Finally the optimal weight on expected inflation decreases with the interest rate if $s_1 < \frac{\phi_2}{\psi_1(\phi_2+\phi_4+\phi_6)}$.

$$\frac{df_\pi}{di_t} = \frac{\alpha_i' \kappa \rho \lambda \left(\phi_2 - \psi_1 s_1 \left(\phi_2 + \phi_4 + \phi_6\right)\right)}{\left[\rho \lambda \left(\phi_6 + \phi_4 + \phi_2 \left(1 - \alpha_t\right)\right)\right]^2}$$

The intuition is again the same as in the previous cases. Note that the interest rate effect and the house price effect tend to offset the other two, since $\alpha_\pi' > 0$, $\alpha_y' > 0$ and $\alpha_i' < 0$, $\alpha_q' < 0$, and the strength of each effect crucially depends on the sensitivities α_π', α_y', α_i' and α_i'.

2.5.2.2 The optimal weight on the output gap

House price changes have an impact on the share of constrained agents and therefore on the optimal weight on the output gap.

$$\frac{df_y}{dq_t} = \frac{\alpha_q' \rho \left[\phi_2 \phi_3 x_3 - x_2 \left(\phi_1 \left(\phi_4 + \phi_6\right) - \phi_2 \phi_5\right)\right]}{\left[\phi_6 + \phi_4 + \phi_2 \left(1 - \alpha_t\right)\right]^2}$$

The intuition is analogous to the case where either all the young are constrained or no one is constrained. The weight on output decreases with the house price if the share of consumption in aggregate income when middle-aged is small, as defined by $x_2 < \frac{\phi_2\phi_3x_3}{\phi_1(\phi_6+\phi_4)-\phi_2\phi_5}$[17]. Higher house prices reduce the proportion of constrained agents. On the one hand these newly unconstrained respond to interest rate changes, which allows to achieve a given reduction in the output gap with a smaller interest rate increase. On the other hand, the newly unconstrained also react to their expected future consumption, which increases with x_2. This effect calls for a stronger interest rate increase to bring down the output gap by a given amount. However, if this effect is small, the first effect dominates and a weaker interest rate response is required.

The optimal weight on the output gap reacts to an increase in the output gap as follows.

$$\frac{df_y}{dy_t} = \frac{\alpha_y'\rho\left[\phi_2\phi_3x_3 - x_2\left(\phi_1\left(\phi_4+\phi_6\right)-\phi_2\phi_5\right)\right]}{\left[\phi_6+\phi_4+\phi_2\left(1-\alpha_t\right)\right]^2}$$

which is positive if $x_2 < \frac{\phi_2\phi_3x_3}{\phi_1(\phi_6+\phi_4)-\phi_2\phi_5}$. The intuition is the same as above.

The optimal weight on output changes with respect to expected inflation and the interest rate in an analogous manner.

$$\frac{df_y}{d\left(E_t\pi_{t+1}\right)} = \frac{\alpha_\pi'\rho\left[\phi_2\phi_3x_3 - x_2\left(\phi_1\left(\phi_4+\phi_6\right)-\phi_2\phi_5\right)\right]}{\left[\phi_6+\phi_4+\phi_2\left(1-\alpha_t\right)\right]^2}$$

$$\frac{df_y}{di_t} = \frac{\alpha_i'\rho\left[\phi_2\phi_3x_3 - x_2\left(\phi_1\left(\phi_4+\phi_6\right)-\phi_2\phi_5\right)\right]}{\left[\phi_6+\phi_4+\phi_2\left(1-\alpha_t\right)\right]^2}$$

2.5.2.3 The optimal weight on house prices

The optimal weight on house prices is a function of house prices themselves.

$$\frac{df_q}{dq_t} = \frac{\alpha_q'\psi_2b\left(\phi_2+\phi_4+\phi_6\right)}{\left[\phi_6+\phi_4+\phi_2\left(1-\alpha_t\right)\right]^2} < 0$$

This is because with rising house prices fewer agents are constrained, who don't react to house prices. In this case consumption and the output do increase. After all, becoming unconstrained means that consumption of the young has increased up to or beyond the optimal level of consumption. However, this increase in consumption is now captured by the increase in the output gap. Therefore a separate response to house prices is not warranted. The difference lies in the coefficients on house prices and the output gap. Pressure on the output gap due to a wealth effect from house prices requires a slightly different response than pressure due to an increase in expected future income and consumption.

Furthermore, the optimal weight on house prices increases with expected

[17] Remember that since $x_2 \geqq 0$, it must hold that $\phi_1\left(\phi_4+\phi_6\right) > \phi_2\phi_5$.

inflation.

$$\frac{df_q}{d\left(E_t\pi_{t+1}\right)} = \frac{\alpha'_\pi \psi_2 b\left(\phi_2+\phi_4+\phi_6\right)}{\left[\phi_6+\phi_4+\phi_2\left(1-\alpha_t\right)\right]^2} > 0$$

This is because more agents are constrained, who react to house price increases. The optimal weight also increases with the output gap

$$\frac{df_q}{dy_t} = \frac{\alpha'_y \psi_2 b\left(\phi_2+\phi_4+\phi_6\right)}{\left[\phi_6+\phi_4+\phi_2\left(1-\alpha_t\right)\right]^2} > 0$$

and decreases with the interest rate. A higher interest rate reduces the proportion of constrained agents, who react to changes in house prices.

$$\frac{df_q}{di_t} = \frac{\alpha'_i \psi_2 b\left(\phi_2+\phi_4+\phi_6\right)}{\left[\phi_6+\phi_4+\phi_2\left(1-\alpha_t\right)\right]^2} < 0$$

2.6 Discussion

2.6.1 House prices are affected by the interest rate

A standard present-value model for house prices would predict that the current house price is a function of the real interest rate[18].

$$q_t = \psi_3 E_t q_{t+1} - \psi_4\left(i_t - E_t\pi_{t+1}\right) + \eta_t$$

Then the fall in the share of constrained agents following an interest rate increase is smaller because in addition to the effect of a lower optimal level of consumption house prices fall reducing liquid assets. Conversely, an interest rate decrease leads to a smaller increase in the share of constrained agents because higher house prices compensate for the increased desired consumption level.

$$\alpha'_i|_{\psi_4=0} < \alpha'_i|_{\psi_4>0}$$

Furthermore, for any given inflation expectation, output gap or lagged house price changes the optimal interest rate rule requires a smaller response because any house price increase is immediately dampened by an interest rate increase.

2.6.2 House prices follow an autoregressive process

So far, we have not specified a time-series process for house prices. However, empirically the growth rate of house prices has been found to be fairly strongly autocorrelated (see Englund and Ioannides, 1997; Case and Shiller, 1989, 1990; Meese and Wallace, 1994). We take account of this empirical regularity by also

[18] Assume for simplicity that the expected future change in rents is zero .

considering the following process for house prices

$$q_t = \tau q_{t-1} + \eta_t$$

where q_{t-1} is the the lagged percentage deviation from steady-state, τ is the autocorrelation coefficient and η_t is the house price shock. Then the share of constrained agents becomes

$$\alpha_t^{AR} = F\left(\frac{\phi_1 x_2}{\psi_1}\rho y_t - \frac{\phi_2}{\psi_1}\left(i_t - E_t\pi_{t+1}\right) - \frac{\psi_2 \tau b}{\psi_1} q_{t-1} - \frac{\psi_2 b}{\psi_1}\eta_t\right)$$

The analysis proceeds as in the case of a random walk for house prices above. New is however that lagged house price changes now appear in the interest rate rule and in the defintion of the share of constrained agents α_t^{AR}. The optimal rule is now

$$i_t = f_\pi E_t \pi_{t+1} + f_y y_t + f_q\left(\tau q_{t-1} + \eta_t\right)$$

Clearly all results derived in the case of a random walk continue to hold with the addition that monetary policy should also react to lagged asset prices with a weight τf_q. The strength of the response to lagged changes in house prices increases with the autoregressive parameter τ. Moreover the weight on past house prices varies with α_t^{AR}, i.e. with expected inflation, the output gap, the interest rate, lagged house prices themselves and the current house price shock. In particular,

$$\frac{df_q}{d\left(q_{t-1}\right)} = \frac{\alpha'_{q_{t-1}}\psi_2 b\left(\phi_2 + \phi_4 + \phi_6\right)}{\left[\phi_6 + \phi_4 + \phi_2\left(1 - \alpha_t\right)\right]^2} < 0$$

The higher lagged house prices the smaller is the required interest rate response to them. The reason is that the higher are lagged house prices the fewer agents are constrained for a given current house price shock. Unconstrained agents don't react to house prices anymore. Moreover, only those who are unconstrained react to interest rate changes. Constrained agents keep on consuming out of their liquid assets. With a higher share of unconstrained agents, a smaller interest rate change is needed to offset the wealth effect from house prices. The effects of expected inflation, output gap and interest rate on the optimal weight on past asset prices follow analogously.

Furthermore, the optimal weights on expected inflation and the output gap are now affected by the presence of lagged house prices.

$$\frac{df_\pi}{dq_{t-1}} = \frac{\alpha'_{q_{t-1}}\tau\kappa\rho\lambda\left(\phi_2 - \psi_1 s_1\left(\phi_2 + \phi_4 + \phi_6\right)\right)}{\left[\rho\lambda\left(\phi_6 + \phi_4 + \phi_2\left(1 - \alpha_t\right)\right)\right]^2}$$

The weight on expected inflation decreases with lagged house prices if $s_1 < \frac{\phi_2}{\psi_1(\phi_2+\phi_4+\phi_6)}$, where the intuition is the same as above. The weight on the output gap is affected in an analogous manner to the case where house price follow a random walk.

2.6.3 Discussion of some model assumptions

The role of bequests In the model it is assumed that the old generation bequeath their houses to their middle-aged descendants. This assumption rules out a wealth effect on consumption of the old. Abolishing housing bequests from the old would introduce another wealth channel into the model. This, however, would be separate from the wealth effect from house prices through relaxing liquidity constraints of the young and would not affect the results derived with regard to the optimal weights on the output gap and expected inflation. However, the weight on house prices themselves would probably increase since more agents would respond to an increase in house prices by expanding consumption. Furthermore, abolishing the bequest motive in terms of the old caring for consumption of their middle-aged descendants would call for specifying consumption of the old in a different way. For example the old could consume their permanent income, which, however would introduce lags of the interest rate, inflation and output. This wouldn't change the results qualitatively while rendering the model more complicated.

Possibility of default of middle-aged While carrying out the analysis above we have maintained the assumption that the middle-aged and the old are always unconstrained. In particular, the assumption was that their respective income is always more than enough to cover desired consumption and desired lending to the young. In addition there is no default on debt. Both assumptions allow to focus solely on the role of house prices as collateral in relaxing liquidity constraints. Default on the part of the middle-aged would have an effect if the repayment was used to finance consumption of the old. Then, a fall in house prices below the contracted loan-to-value ratio would depress consumption of the old in addition to the reduction in consumption by the constrained young. To connect the possibility of default to house prices one could introduce a fourth generation between the middle-aged and the old. The income of the middle-aged might not be sufficient to cover both consumption and repayment of the loan. They would have to roll over their loan by borrowing from the additional generation again with their housing value as collateral, the same mechanism as for borrowing by the young. If house prices fell, the middle-aged wouldn't be able to cover their repayment by a new loan and would default. However, even without appealing to these arguments we have shown that house prices do matter in the optimal conduct of monetary policy.

Distinction between bubble and fundamental price change So far we haven't made any assumption about the source of a house price increase. It could be fundamentally justified or it could be driven by non-fundamental factors. Whether this matters for the model depends on the expectations of the young borrowers and the middle-aged lenders about the persistence of the house price boom. If both expect it to last at least until the next period borrowers and lenders are happy to accept the value of the house as collateral even though at some point in time it might fall considerably. This again results from the role of housing as collateral, which allows to bring forward consumption from later periods. Under this view it doesn't matter whether consumers believe house prices are driven by fundamental or non-fundamental factors.

2.7 Conclusion

In this paper we have derived a wealth effect from house prices through their role as collateral to finance consumption. Housing value serves as a means to bring forward consumption in time without being of intrinsic value. Since house prices vary, so does the value of collateral and therefore the proportion of constrained agents varies too. Furthermore, the share of constrained agents depends on house prices, expected inflation, the output gap and the interest rate. Since constrained and unconstrained agents react differently to house price changes, expected future output gap, expected inflation and interest rate changes, the actual share of constrained agents is important for the weights monetary policy should put on each of these factors when setting interest rates. In sum, the analysis shows that the optimal weights on expected inflation, the output gap and house price changes vary over time, in turn depending on the values for expected inflation, the output gap and house price changes. Therefore house prices do seem to play a role in the optimal response of monetary policy to house prices over and above their effect on aggregate demand and the output gap. This result has been derived without appealing to supply side effects from defaults on debt or the informational content in asset prices about future productivity or inflation.

The model has also demonstrated that it is important where a wealth effect comes from. If it results from relaxed liquidity constraints there is the additional effect on the optimal weights on inflation, output and house prices in the interest rate rule. Therefore we have worked out another factor that is relevant for an appropriate interest rate response in the face of changes in expected inflation, the output gap and house prices.

References

Altissimo, F., E. Georgiu, T. Sastre, M. Valderrama, G. Sterne, M. Stocker, M. Weth, K. Whelan and A. Willman (2005): Wealth and asset price effects on economic activity. ECB Occasional Paper Series No. 29.

Attanasio, O. and M. Browning (1995): Consumption over the life cycle and over the business cycle. *American Economic Review*, Vol. 85, No. 5, 1118-1137.

Attanasio, O. and G. Weber (1994): The UK consumption boom of the late 1980s: Aggregate implications of microeconomic evidence. *The Economic Journal*, Vol. 104, No. 427, 1269-1302.

Bean, C. (2004): Asset prices, financial instability, and monetary policy. *American Economic Review*, Vol. 94, No. 2, Papers and Proceedings, 14-18.

Bernanke, B. and M. Gertler (1999): Monetary policy and asset price volatility. Economic Review of the Federal Reserve Bank of Kansas City (Fourth Quarter), 17-51.

Bernanke, B. and M. Gertler (2001): Should central banks respond to movements in asset prices. *American Economic Review*, Vol. 91, No. 2, Papers and Proceedings, 253-257.

Bordo, M. and O. Jeanne (2002): Boom-busts in asset prices, economic instability, and monetary policy. NBER Working Paper No. 8966.

Borio, C. and P. Lowe (2002): Asset prices, financial and monetary stability: Exploring the nexus. Bank for International Settlements Working Paper No. 114.

Campbell, J. and J. Cocco (2007): Do house prices affect consumption? Evidence from micro data. *Journal of Monetary Economics* 54, 591-621.

Carroll, C. (1997): Buffer-stock saving and the life-cycle/permanent income hypothesis. *Quarterly Journal of Economics*, Vol. 112, No. 1, 1-55.

Carroll, C. (2004): Housing wealth and consumption expenditure. Mimeo. http://econ.jhu.edu/people/ccarroll/papers/FedHouseWealthv2.pdf

Carroll, C., M. Otsuka and J. Slacalek (2006): How large is the housing wealth effect? A new approach. NBER Working Paper No. 12746.

Case, K., J. Quigley and R. Shiller (2001): Comparing wealth effects: The stock market versus the housing market. NBER Working Paper No. 8606.

Case, K. and R. Shiller (1988): The efficiency of the market for single-family homes. *American Economic Review*, Vol. 79, No. 1, 125-137.

Case, K. and R. Shiller (1990): Forecasting prices and excess returns in the housing market. *American Real Estate and Urban Economics Association Journal*, Vol. 18, No. 3.

Cecchetti, S., H. Genberg, J. Lipsky and S. Wadhwani (2000): Asset prices and central bank policy. Geneva Reports on the World Economy 2.

Davis, M. and M. Palumbo (2001): A primer on the economics and time series econometrics of wealth effects, Finance and Economics Discussion Series, No. 2001-9, Federal Reserve Board, Washington D.C.

Deaton, A. (1991): Saving and liquidity constraints. *Econometrica*, Vol. 59, No. 5, 1221-1248.

Englund, P. and Y. Ioannides (1997): House price dynamics: An international empirical perspective. *Journal of Housing Economics*, Vol. 6, 119-136.

Filardo, A. (2004): Monetary policy and asset price bubbles: Calibrating the monetary policy trade-offs. Bank for International Settlements Working Paper No. 155.

Gourinchas, P. and J. Parker (2002): Consumption over the life cycle. *Econometrica*, Vol. 70, No. 1, 47-89.

Greenspan, A. (1999): General Discussion: monetary policy and asset price volatility, in: New Challenges for Monetary Policy. Symposium sponsored by the Federal Reserve Bank of Kansas City Jackson Hole, Wyoming.

Greenspan, A. (2004): Risk and uncertainty in monetary policy. Remarks at the meeting of the American Economic Association, San Diego, California, January 3, 2004.

Greenspan, A. and J. Kennedy (2005): Estimates of home mortgage originations, repayments, and debt on one-to-four-familiy residences, Finance and Economics Discussion Series, No. 2005-41, Federal Reserve Board, Washington D.C.

Greenspan, A. and J. Kennedy (2007): Sources and uses of equity extracted from homes. Finance and Economics Discussion Series, No. 2007-20, Federal Reserve Board, Washington D.C.

Gruen, D., M. Plumb and A. Stone (2005): How should monetary policy respond to asset price bubbles? *International Journal of Central Banking*, Vol. 1, No. 3, 1-31.

Iacoviello, M. (2004): Consumption, house prices and collateral constraints: A structural econometric analysis. *Journal of Housing Economics*, Vol. 13, No. 4, 304-320.

Iacoviello, M. (2005): House prices, borrowing constraints and monetary policy in the business cycle. *American Economic Review*, Vol. 95, No. 3, 739-764.

Illing, G. and U. Klüh (2005): Vermögenspreise und Konsum - Neue Erkenntnisse, amerikanische Erfahrungen und europäische Herausforderungen. *Perspektiven der Wirtschaftspolitik* Vol. 6, No. 1, 1-22.

Japelli, T. (1990): Who is credit constrained in the US economy? *Quarterly Journal of Economics*, Vol. 105, No. 1, 219-234.

Kent, C. and P. Lowe (1997): Asset price bubbles and monetary policy. Research Discussion Paper 9709, Reserve Bank of Australia.

Kontonikas, A. and A. Montagnoli (2006): Optimal monetary policy and asset price misalignments. *Scottish Journal of Political Economy*, Vol. 53, No. 5, 636-654.

Ludvigson, S. (1999): Consumption and credit: A model of time-varying liquidity constraints. *Review of Economics and Statistics*, Vol. 81, No. 3, 434-447.

Lustig, H. and S. van Nieuwerburgh (2006): Can housing collateral explain long-run swings in asset returns? NBER Working Paper No. 12766.

Meese, R. and N. Wallace (1994): Testing the present value relation for housing prices: Should I leave my house in San Francisco? *Journal of Urban Economics*, Vol. 35, 245-266.

Monacelli, T. (2006): Optimal monetary policy with collateralized household debt and borrowing constraints. NBER Working Paper No. 12470.

Muellbauer, J., A. Murphy, M. King and M. Pagano (1990): Is the UK balance of payments sustainable? *Economic Policy*, Vol. 5, No. 11, 347-395

Pesaran, M. and R. Smith (1995): The role of theory in econometrics, *Journal of Econometrics*, Vol. 67, 61-79.

Ortalo-Magné, F. and S. Rady (1999): Boom in, bust out: Young households and the housing price cycle. *European Economic Review*, Vol. 43, 755-766.

Ortalo-Magné, F. and S. Rady (2006): Housing market dynamics: On the contribution of income shocks and credit constraints. *Review of Economic Studies*, Vol. 73, 459-485.

Romer, D. (2001): *Advanced macroeconomics*, Berkeley, McGraw-Hill, 2nd edition.

Smets, F. (1997): Financial asset prices and monetary policy: Theory and evidence, Bank for International Settlements Working Paper No. 47.

Walsh, C. (2003): *Monetary theory and policy*, Cambridge, MIT Press, 2nd edition.

Appendix 2.A Derivation of the optimal interest rate rule

The solution for optimal monetary policy are expressions for the output gap and inflation in only the state variables. Under discretion e_t is the only relevant state variable such that a conjectured solution is of the form

$$y_t = \delta e_t \tag{2.14}$$

From the optimality condition (2.10)

$$\pi_t = -\frac{\lambda}{\kappa} y_t$$

it follows

$$\pi_t = -\frac{\lambda}{\kappa} \delta e_t$$

Plugging this into the Phillipscurve (2.8) yields

$$y_t = \frac{\beta \delta \rho - \frac{\kappa}{\lambda}}{1 + \frac{\kappa^2}{\lambda}} e_t \tag{2.15}$$

Equating coefficients from (2.14) and (2.15) results in

$$\delta = \frac{-\kappa}{\kappa^2 + \lambda (1 - \rho\beta)}$$

Consequently,

$$y_t = \frac{-\kappa}{\kappa^2 + \lambda (1 - \rho\beta)} e_t \tag{2.16}$$

and

$$\pi_t = \frac{\lambda}{\kappa^2 + \lambda (1 - \rho\beta)} e_t \tag{2.17}$$

To arrive at the optimal interest rule use (2.16) and (2.17) together with the AR(1) process for the cost push shock in the IS curve (2.6).

Chapter 3

U.S. stock prices and moral hazard: Did the Fed contribute to the bubble in the late 1990s?

3.1 Introduction

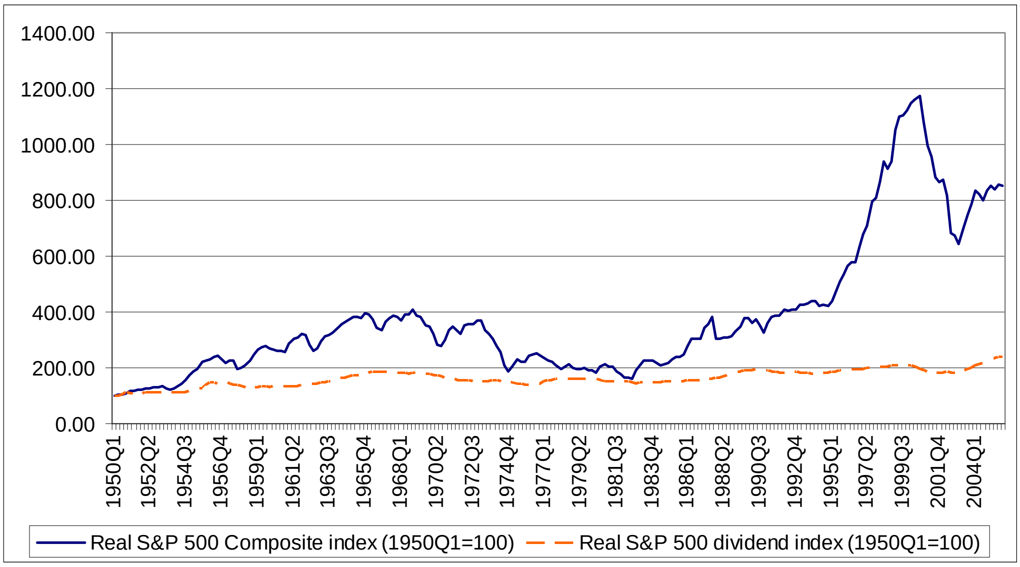

Figure 3.1: Real S&P 500 (solid line) and real S&P 500 dividend payments index (dashed line).
Notes: Quarterly data, 1950 Q1 to 2005 Q4. Source: Standard and Poor's.

Figure (3.1) shows the quarterly real S&P 500 stock price index along with the S&P 500 dividend payments index from 1950 to 2005. What stands clearly out is the huge peak in stock prices in the late 1990s. The aim of this paper is to test whether the Fed under its chairman Alan Greenspan[1] indirectly contributed

[1] Alan Greenspan served as chairman of the Fed from 11 August 1987 to 31 January 2006

to the bubble. The Fed may have done so because investors believed the Fed will bail them out in case of a stock market crash by injecting liquidity and stabilizing stock prices. Investors may have come to have this belief because the Fed has acted accordingly after the stock market crash on the Black Monday of October 1987 and after the LTCM crisis in September 1998[2]. This hypothesis, which is known as the Greenspan-put, has been supported by leading academics and the media.

- Cecchetti et al (2000): "Many analysts have expressed concern that central banks may have created moral hazard by creating expectations that they would take remedial policy action if asset prices fall."
- Mussa (2003): "To this was added the market perception reinforced by the Fed's response to the LTCM crisis, that US monetary policy would act aggressively to countervail any sharp sell-off in equity markets – making investment in equities appear to have some characteristics of a one-way bet."
- Filardo (2004): "... investors are likely to take too much risk during the good times because investors may perceive the monetary authority as providing free downside risk insurance in the case of bad times – in the language of options, the monetary authority is offering an unpriced put."
- Mishkin and White (2003): "A fourth problem with too much focus on the stock market is that it may create a form of moral hazard. Knowing that the central bank is likely to prop up the stock market if it crashes, the markets are then more likely to bid up stock prices. This might help facilitate excessive valuation of stocks and help encourage a stock market bubble that might crash later... "
- Borio and Lowe (2003): "Moreover, reaction functions that are seen to imply asymmetric responses, lowering rates or providing ample liquidity when problems materialize but not raising rates as imbalances build up, can be rather insidious in the longer run. They promote a form of moral hazard that can sow the seeds of instability... "
- Financial Times (2001): "It's official: there is a Greenspan put option. (...) By showing investors he [Greenspan] will rescue them come what

[2] "...central banks do not respond to gradually declining asset prices. We do not respond to gradually rising asset prices. We do respond to sharply reduced asset prices, which will create a seizing up of liquidity in the system." Greenspan (1999)

"...Instead of trying to contain a putative bubble by drastic actions with largely unpredictable consequences, we chose, as we noted in our mid-1999 congressional testimony to focus on policies 'to mitigate the fallout when it occurs and, hopefully, ease the transition to the next expansion'". Greenspan (2004) may, he is encouraging excessive risk-taking and the formation of future bubbles."

- The Economist (1999): "In recent years, Mr Greenspan has been taking a big risk by not having tightened policy when he first thought a bubble might be forming, or subsequently. There were plenty of signs of overheating, such as rampant consumer borrowing, that he could have used to justify higher interest rates. His second risky move was to cut rates three times last autumn in response to financial turmoil, when policy was already lax, and then to fail to take back this easing as soon as financial markets had stabilised. The Fed has thereby fostered the impression that it will slash interest rates when share prices fall sharply, but not increase rates when they shoot up. This apparent asymmetry has created a form of moral hazard that encourages investors to take bigger risks."

  and

  "Mr Greenspan's confidence that he can use monetary policy to prevent a deep recession if share prices crash exposes an awkward asymmetry in the way central banks respond to asset prices. They are reluctant to raise interest rates to prevent a bubble, but they are quick to cut rates if financial markets tremble. Last autumn, in the wake of Russia's default and a slide in share prices, the Fed swiftly cut rates, saying it wanted to prevent a credit crunch. As a result, share prices soared to new highs. The Fed has inadvertently created a sort of moral hazard. If investors believe that monetary policy will underpin share prices, they will take bigger risks."

- The Economist (2006): "If the Fed always cuts interest rates when asset prices tumble, but never raises them when they soar, then investors will be encouraged to take bigger risks."

Of course no one claims that moral hazard induced by monetary policy actions was solely responsible for the surge in stock prices in the late 1990s. Shiller (2001) neatly summarizes a number of factors that may have played a role in pushing up stock prices: the internet as a new technology, the spreading of the capitalist system in the world (especially China opening the economy to market oriented ideas), management and employee stock options, capital gains tax cut by a republican congress, the baby boom after WW II[3], and various psychological factors such as increased positive coverage of business by the media or the decline of inflation. Next to these factors, some authors put forward reasons

[3] Birth rates in 1946-1966 were very high. These people now save for their retirement by investing in stocks. In addition demand for goods is generally high with a larger population which results in higher profits of firms and thus a high price-earnings ratio (Shiller, 2001).

such as the decline in macroeconomic volatility, which reduces the risk-premium and raises stock prices (Lettau, Ludvigson and Wachter, 2006). While these explanations surely deserve attention, the moral hazard argument is especially interesting because it has a direct bearing on the more general discussion about whether monetary policy should react to asset prices. Opinions on this matter can roughly be grouped around three persons. Ben Bernanke, the current chairman of the Fed, believes that asset prices should play no role in monetary policy making over and above their impact on the inflation forecast (Bernanke and Gertler, 2001). In contrast, Stephen Cecchetti argues that monetary policy can and should take asset prices into account when setting interest rates and react preemptively to a developing bubble. The reason is that once asset prices crash output might decline and as a result consumer price inflation might be affected. A sufficiently forward looking central bank might wish to avoid the extra volatility in output and inflation resulting from the asset price crash (Cecchetti et al., 2000). Finally, Alan Greenspan holds the view that it is impossible to identify asset price misalignments in the first place. Consequently, a central bank cannot react to asset price movements. All it can do is to stand ready and limit the damage after a stock market crash by providing liquidity to the market (Greenspan, 1999; Greenspan, 2004)[4]. Not only disagree Cecchetti et al. (2000) with the view that asset price bubbles are impossible to identify[5], but Greenspan's position might have lead to the alleged moral hazard behaviour on the part of investors. If that was true monetary policy-makers might rather opt for acting preemptively against rapidly rising asset prices to avoid both the moral hazard problem and any adverse effects on output and inflation should the bubble burst.

The remainder of the paper is structured as follows. In the next section we outline our empirical strategy, section 3 gives a literature overview, section 4 describes the data, section 5 presents unit root tests and cointegration analysis, section 6 reports results from a state-space estimation to identify asset price misalignments for the years 1950 to 2005. The seventh section uses theoretical models to derive measures of moral hazard in monetary policy and empirically tests their influence on U.S. stock prices. Section 8 concludes.

3.2 Empirical strategy

To be able to analyse factors that might have had an impact on stock price bubbles, the bubble must be identified first. In the literature the term bubble

[4] See footnote 2.

[5] Cecchetti et al. (2000) argue that a central bank uses the output gap to gauge inflation, which requires a judgement about unobserved output. Analogously it shouldn't be impossible to arrive at a judgement about the fundamental value of asset prices.

is mostly used for any kind of variation in asset prices that can't be explained by fundamentals, as derived from some model. Following this definition of a bubble we start from the standard present value model. The stock price is the discounted sum of expected future dividend payments. Instead of imposing the transversality condition we explicitly allow for a non-fundamental term in the stock price equation, which we call bubble. This formulation allows to use unit root and cointegration tests as a preliminary check for bubbles. After having established that bubbles can't be ruled out by these tests we cast the problem in terms of a state-space model and use the Kalman filter technique to get an explicit estimate of the unobserved bubble term.

In the second stage of our analysis the identified bubble term is used to test whether investor moral hazard induced by the Fed's past actions had any impact on it. To do so we rely on two theoretical models that address the problem of moral hazard in U.S. monetary policy. We construct various measures of moral hazard on the basis of these models and test whether they had an impact on the bubble term itself or whether they are significant in the stock price equation.

Our results are that unit root and cointegration tests indicate the presence of a bubble and we are indeed able to identify a statistically significant bubble component in U.S. stock prices in the late 1990s using the state-space approach. Regarding the measures of moral hazard none of them turns out to have a significant impact on either the bubble term itself or on stock prices directly.

There are three basic criticisms to our approach. First, as Cogley (1999) argues, researchers trying to detect a bubble, e.g. at central banks, have difficulties in observationally distinguishing a bubble from an omitted unobserved fundamental. This is an important criticism which in principal applies to our approach. However, the crucial difference is that we take the identification of the bubble one step further and use it to test one specific hypothesis, the Greenspan put hypothesis. That is, we treat the unobserved moral hazard behaviour on the part of investors as the omitted variable and test whether it had any impact on the identified bubble. In addition to that, Cecchetti et al's (2000) argument applies. Estimating the output gap requires a judgement about the level of potential output, which is common practice at central banks. Equally, identifying a bubble requires a judgement about the fundamentals, which has its problems but which is not impossible. Second, our test for a bubble is principally also a test of the adequacy of the present value model. However, this criticism applies only if one uses the present value to reject the presence of the bubble. Then, one wouldn't know whether the rejection is due to there truly being no bubble or due to the model being inadequate. In contrast, not rejecting a bubble poses no problem in this regard. However, third, rejecting the moral hazard hypothesis might mean the measures of moral hazard are right but have truly no statistically significant impact. Or the measures of moral hazard are false. Since moral hazard behaviour is not observable we must rely on indirect measures. The next best option is then to use theory to arrive at those measures, which is our approach. At the very least, our results indicate that the predictions of the theoretical models of Miller, Weller and Zhang (2002) and Illing (2001) are not confirmed by the data.

3.3 Related literature

There are various approaches in the literature to testing for asset price bubbles[6]. The starting point is mostly the present value model of stock prices. The earliest tests by Shiller (1981) and LeRoy and Porter (1981) were variance bounds test. Their intuition is that the ex-ante expected stock price should be less volatile than the ex-post realised value calculated from realised dividend payments because it contains the forecast error of dividends. This implies a restriction on the variances of the observed prices at time t and their ex-post realised values which can be tested. Their finding is that the variance restriction is violated and the existence of bubbles can't be ruled out. Of course, all tests for bubbles based on the present value model are at the same time tests of the present value model itself. West (1987) proposed a way to overcome this difficulty. The idea is to obtain the parameters of the present value model by estimating an Euler equation and an autoregressive process for dividends. Misspecification tests are applied to ensure the validity of the estimates. They can then be used to re-construct the relation between the stock price and fundamentals. In a second step stock prices can be estimated using the present value model. If the two estimated relationships differ it is possible to distinguish model misspecification and the presence of a bubble. West can't rule out the presence of a bubble either. Another strand of the literature exploits unit root and cointegration characteristics of the present value model taking into account possible unobservables. Notably Diba and Grossman (1987, 1988a, 1988b) follow this approach and see whether they can rule out an explosive rational bubble in stock prices[7]. After applying various unit root and cointegration tests they conclude that stock prices don't have an explosive rational bubble component. Psaradakis, Sola and Spagnolo (2001) examine wholesale prices and the money supply during the German hyperinflation. They apply a specific unit root test and find an explosive root in their data. This approach has been criticised by Evans (1991) who presents an example of a periodically collapsing bubble which never bursts. By subjecting a simulated periodically collapsing bubble to Diba and Grossman's (1988) test

[6] Gurkaynak (2005) provides a thorough overview of empirical tests for asset price bubbles.
[7] This approach will be discussed in detail and applied below.

he shows that unit roots tests fail to detect this kind of bubble[8]. The literature on markov-switching processes in stock prices and dividends tries to overcome this problem. Hall, Psaradakis and Sola (1999) use a markov-switching model to identify periods when asset prices are in an explosive regime while fundamentals are not. They apply it to consumer prices, the money supply and the exchange rate and identify periods of rational explosive bubbles in consumer prices and the exchange rate. Psaradakis, Sola and Spagnolo (2004) use a Markov error-correction model to identify periods of collapsing bubbles and conclude against their existence[9]. Instead of concentrating on modelling periodic collapses of bubbles, Wu (1995, 1997) focuses on the fact that the bubble is unobserved by the econometrician and proposes a Kalman filter approach to testing for bubbles. The Kalman filter allows for the estimation of an unobserved variable within a state-space model, e.g. the bubble term in the present value model. Applying this technique to U.S. exchange rate data he finds no support for the existence of bubbles. However, testing for a bubble in U.S. stock prices in the S&P 500 he can identify stock price bubbles. Pastor and Veronesi (2006) employ a standard stock valuation model and show that once one takes into account uncertainty about future dividend growth the observed values of the NASDAQ index can be replicated by calibrating their model. Their argument is that since the price-dividend ratio is a convex function of the growth rate of dividends it is increasing in the uncertainty about future dividend growth. To the extent that the late 1990s in the U.S. were characterised by high uncertainty about future firm profits, especially in the information and communications sector, their model can explain high stock prices in the NASDAQ without recurring to a bubble.

In contrast there is hardly any literature on moral hazard and monetary policy. Only four papers address the problem of theoretically modelling moral hazard in monetary policy. Illing (2001) shows why it may be rational for a central bank to react asymmetrically to asset price movements building on a framework by Allen and Gale (2000). Because it is costly to let highly leveraged firms go bankrupt on a large scale the central bank has an incentive to inject liquidity in case of an aggregated shock. This incentive is higher, the higher the leverage in the economy. Rational investors will anticipate the resulting capital gain from the reduced real debt burden and include it in their stock valuation. In a related paper Cao and Illing (2007) analyse the incentives for financial market actors to free-ride on liquidity provision by the central bank,

[8] Taylor and Peel (1998) use a unit root test that is robust to periodically collapsing components in stock prices and reject the hypothesis of a stock price bubble in the U.S. Sarno and Taylor (1999) apply the same test to East Asian stock price indices and confirm the existence of bubbles there.

[9] Furthermore, Froot and Obstfeld (1991) propose a model where the stock price bubble is a function of dividends, which is called an intrinsic bubble. Their own test and a test by Driffil and Sola (1998) yields ambiguous results regarding the presence of intrinsic bubbles.

which can lead to excessive risk-taking. Furthermore, Sauer (2007) shows in a model of optimal liquidity provision by the central bank that the anticipation of which by investors results in more investment into the possibly less liquid asset than without the central bank intervention. In addition, Miller, Weller and Zhang (2002) can explain the observed low risk-premium in the late 1990s by incorporating the value of an implicit insurance of investors against downside risk. Related but not modelling stock price misalignments is the paper by Borio and Lowe (2002). They study ex-ante indicators of financial crises and show that the deviation of the ratio of credit to GDP from trend is a fairly good indicator of financial crises.

Finally, to the best of our knowledge, there is no paper which tries to evaluate empirically the hypothesis of moral hazard in monetary policy.

In the following we set up the present value model of stock prices and apply unit root and cointegration tests on U.S. stock price data. After being able to reject the null hypothesis of no bubble we proceed to estimating a state-space model using the Kalman filter which allows to obtain an actual series with confidence bands for the bubble term and use measures of moral hazard to test the moral hazard hypothesis.

3.4 Data

The stock price index is the S&P 500 composite index from 1950 Q1 to 2005 Q4. The dividend series is the S&P 500 dividend series from 1950 Q1 to 2005 Q4, backed out from the S&P 500 dividend yield. Because it is not seasonally adjusted we also used the seasonally adjusted U.S. net corporate dividend payments from 1950 Q1 to 2005 Q4 as a cross-check. It comes from the National Income and Product Accounts Table 1.12 of the Bureau of Economic Analysis and includes dividend payments by domestic financial and nonfinancial firms, the farm sector and foreign subsidiaries received by U.S. residents. It is a broader measure for dividends than associated with the S&P 500 composite index. All series are deflated by the seasonally adjusted U.S. consumer price index. The real interest rate is the annualized three-month U.S. treasury bill rate minus the CPI based inflation rate.

Data for the various measures of moral hazard are constructed from different sources. The stock market crash probability is Shiller's Crash Confidence Index from a survey among institutional investors available on his website[10]. It is the percentage of respondents who think that the probability of a stock market crash in the next six months is less than 10%. The data are collected semi-annually

[10] For more information on Shiller's investor confidence indices: http://icf.som.yale.edu/confidence.index/

from October 1989 to April 2001 and monthly afterwards. To arrive at quarterly data to make frequencies match, we have linearly interpolated Shiller's survey data from October 1989 to April 2001 and averaged from July 2001 onwards. Miller, Weller and Zhang's (2002) measure of moral hazard has been constructed by taking the ratio of the level of current dividends to their level in 1987 Q4, immediately after the crash, and to their level at 79% of the stock price peak in 1998 Q2. Real credit growth is the real growth of total U.S. non-federal debt outstanding deflated by the CPI. The debt gap measure has been constructed by applying the HP-filter to the ratio of total non-financial sector debt outstanding to seasonally adjusted GDP with a smoothing parameter of 1600.

3.5 The present value model and testable implications for bubbles

As a preliminary test of bubbles in U.S. stock prices we follow Diba and Grossman (1988a) who use the standard present value model to derive testable implications for bubbles. They explicitly allow for an unobserved variable that might influence the stock price over and above dividends and a possible bubble. In their analysis they use data up to 1986 and can rule out rational explosive bubbles. In contrast, extending the data range up to 2005 we can't rule out the existence of either an unobserved variable influencing stock prices or the presence of a bubble.

Consider the stock price according to the present value model

$$P_t = (1+R)^{-1} E_t \left(P_{t+1} + D_t + u_t\right)$$

where P_t is the stock price at the beginning of period t, D_t is the dividend paid during period t, R is the constant real interest rate[11] and u_t is a variable that is unobserved by the researcher but taken into account by market participants. The fundamental stock price F_t is the discounted sum of expected future dividends D_t plus the unobserved variable u_t.

$$F_t = \sum_{i=0}^{\infty} (1+R)^{-i} E_t(D_{t+i} + u_{t+i})$$

The general solution to the stock price equation is

$$P_t = \sum_{i=0}^{\infty} (1+R)^{-i} E_t(D_{t+i} + u_{t+i}) + B_t$$

[11] Assuming a constant real interest rate is standard in the literature. As a check we ran all tests allowing for a time-varying real interest rate, and the results didn't change qualitatively.

where B_t is the bubble term and obeys

$$E_t B_{t+1} = (1 + R)\, B_t$$

Note that since $1+R > 1$ the present value model predicts explosive bubbles, i.e. the bubble should grow at the rate of real interest. Given this setup Diba and Grossman (1988a) derive testable implications for the presence of a bubble in stock prices. If there are no bubbles and if, in addition, the first differences of the unobservable and the first differences of dividends are stationary, then the first differences of stock prices should be stationary too. Moreover, if there are no bubbles and the unobservable is stationary in levels and dividends are stationary in first differences then stock prices and dividends should be cointegrated of order (1,1). More formally,

$$\text{If } B_t = 0 \ \forall t \text{ and } \Delta u_t \sim I(0) \text{ and } \Delta D_t \sim I(0), \text{ then } \Delta P_t \sim I(0) \tag{3.1}$$

$$\text{If } B_t = 0 \ \forall t \text{ and } u_t \sim I(0) \text{ and } \Delta D_t \sim I(0), \text{ then } \begin{pmatrix} P_t \\ D_t \end{pmatrix} \sim CI(1,1) \tag{3.2}$$

Confirming these results would be evidence against the existence of rational bubbles. Rejecting them, however, doesn't necessarily point to the existence of bubbles since, in the first case, it could be that the first differences of the unobservable are non-stationary, while in the second case, the level of the unobservable could be non-stationary.

In the following we report results of unit root tests on stock prices and dividends, as well as results of cointegration tests on stock prices and dividends[12]. Augmented Dickey-Fuller tests have been carried out on the real stock price in levels and differences and the same for real dividends. We included a trend in the levels regression on the stock price and dividends but excluded it in the regression of first differences as well as in the cointegrating regression. An intercept was always included. The lag length was chosen on the basis of the Akaike criterion, the Schwarz Bayesian Criterion and the LR-ratio. In most cases the three criteria agreed on the optimal lag length. Where they didn't agree all suggested lag lengths have been tried. The results were qualitatively the same. The eighth line in table 3.1 contains the values of the t-statistics on the coefficient ρ in the ADF regression with the corresponding 5% critical values in line nine.

The results show that both the price series and the dividend series are $I(1)$ in levels and $I(0)$ in first differences. Column six indicates that the stock prices and dividends are not cointegrated at the 5% level. The predictions of the present

[12] The tests are applied to the levels of all variables. For a logarithmic version including a time-varying real interest rate see the appendix.

	ADF regression $\Delta y_t = \mu + \gamma t + \rho y_{t-1} + \sum_{i=1}^{n} \beta_i \Delta y_{t-i} + \epsilon_t$ $H_o : \rho = 0$, unit root in y_t $T = 224$				
y_t	P_t	ΔP_t	D_t	ΔD_t	$\binom{P_t}{D_t}$
		$\gamma = 0$		$\gamma = 0$	$\gamma = 0$
n^o of lags	4	3	5	5	3
t-statistic on ρ	-1.107	-10.864^*	0.973	-8.558^*	-2.207
5% critical value	-3.423	-2.876	-3.423	-2.876	-3.380

Table 3.1: ADF unit root and cointegration tests on the real stock price and dividends

value model in (3.1) are clearly confirmed, while (3.2) is rejected. The main result to take away is that the test clearly rejects the prediction that a bubble should be explosive. Moreover, the results indicate that the unobservable u_t is not $I(0)$ in levels but more likely to be $I(1)$. This means that there is quite likely an unobserved variable rather than an explosive bubble component that influences stock prices.

To further investigate the possibility of an explosive bubble component in stock prices we employ another test which has been proposed by Bhargava (1986). Next to a test statistic for the null hypothesis of a unit root versus stationarity he provides a direct test of the null of a unit root against an explosive alternative. The test for the null of a simple random walk against the stationary alternative is based on the statistic

$$R_1 = \frac{\sum_{t=2}^{T} (y_t - y_{t-1})^2}{\sum_{t=1}^{T} (y_t - \bar{y})^2}$$

where $\bar{y}$ is the sample average. One rejects the null of a random walk in favour of stationarity in y_t if R_1 becomes larger than some critical value. This is intuitive because the denominator of R_1 grows much faster for a non-stationary series than for a stationary one. The test for the null of a simple random walk against the explosive alternative is based on the statistic

$$N_1 = \frac{\sum_{t=2}^{T} (y_t - y_{t-1})^2}{\sum_{t=2}^{T} (y_t - y_1)^2}$$

One rejects the null of a random walk in favour of the explosive alternative in y_t if N_1 becomes smaller than some critical value. Intuitively, this is because for an explosive series the denominator of N_1 grows much faster than for a simple random walk. R_2 and N_2 work similarly for the null of a random walk with drift.

	Bhargava test for stationarity $H_o : y_t$ is random walk $H_1 : y_t$ is stationary $T = 224$		
y_t	P_t (ΔP_t)	D_t (ΔD_t)	Residuals from cointegrating regression (Δresiduals)
Bhargava test statistic	$R_2 = 0.0217$ $(R_2 = 1.1654^*)$	$R_2 = 0.0140$ $(R_2 = 0.8647^*)$	$R_1 = 0.0170$ $(R_1 = 1.3000^*)$
5% critical value	0.1597	0.1597	0.1194

Table 3.2: Bhargava tests for stationarity on the real stock price, dividends and cointegration residuals.
Notes: Values for first differences in parentheses. Asterisks denote rejection of the null. Test statistic must exceed critical value.

	Bhargava test for explosive roots $H_o : y_t$ is random walk $H_1 : y_t$ is explosive $T = 224$		
y_t	P_t	D_t	Residuals from cointegrating regression
Bhargava test statistic	$N_2 = 0.0168$	$N_2 = 0.0128$	$N_1 = 0.0120$
5% critical value	0.0097	0.0097	0.0027

Table 3.3: Bhargava tests for explosive roots on the real stock price, dividends and cointegration residuals.
Notes: Asterisks denote rejection of the null. Test statistics must be lower than critical value.

Bhargava (1986) tabulates critical values, however only up to a sample size of 100. Since our sample size in this case is 224 we calculated the corresponding 5% critical value by Monte Carlo simulations[13]. In table 3.2 the null hypothesis is that a variable follows a random walk against the stationary alternative. The null is rejected for test statistics exceeding their critical value. In our case we can't reject the null of a random walk for the stock price, dividends and the residuals from the cointegrating regression. This supports the view that the failure of stock prices and dividends to cointegrate is due to some unobserved $I(1)$ variable rather than an explosive rational bubble.

[13] The simulations were cross-checked by first replicating those critical values tabulated by Bhargava (1986). Simulations were carried out running 100000 replications.

Table 3.3 presents results for tests of the null of a random walk against the explosive alternative. The null is rejected for test statistics below the critical value. In this case the null of a random walk can't be rejected for the stock price, the dividend series and the residual.

Overall, the tests confirm the absence of rational explosive bubbles in stock prices, while at the same time indicating the presence of some unobserved variable that follows a random walk. However, what the test doesn't provide information about is at what times the unobserved variable had an impact on stock prices and whether this influence was economically and statistically significant. Moreover, there is another important caveat about using unit root and cointegration tests to identify rational explosive bubbles that was put forward by Evans (1991). He has shown the theoretical possibility of periodically collapsing bubbles. Rational bubbles would then only appear explosive during their expansion, while the subsequent collapse could make the bubble look like an $I(1)$ variable or stationary. This would mean that tests based on random walk and cointegrating properties wouldn't detect a bubble since they focus on the explosive characteristic.

Summing up, the unit root/cointegration approach has two shortcomings. First, while it confirmes the presence of some unobserved variable in U.S. stock prices, rational bubbles can be periodically collapsing and might therefore appear to be integrated of order one instead of explosive as theory suggests (Evans, 1991). Thus, periodically collapsing bubbles cannot be ruled out. Second, it doesn't provide any information about the level or significance of the bubble at different points in time. We are especially interested whether there was a bubble in the late 1990s. Thus to further investigate the presence of bubbles we cast the present value model in a state-space representation and employ the Kalman filtering technique to get an actual estimate of the size and significance of the bubble.

3.6 Estimation of a state-space model

In the previous section it has been argued that pure unit root and cointegration tests are unable to identify periodically collapsing bubbles. The state-space model is better suited to find a possible bubble. It uses the Kalman filtering technique to arrive at an actual time-series estimate of a possible bubble, i.e. it can provide information about the size and significance of a possible bubble. This estimate can then be used to test for determinants. Further advantages of the state-space approach are that it is readily applied to the present value model, it is intuitive and computationally feasible.

3.6.1 The present value model in state-space representation

Next, we formulate the present value model in its logarithmic version to fit it into a state-space representation. Consider again the stock price

$$P_t = \frac{E_t\left(P_{t+1} + D_t\right)}{1 + R_t}$$

where P_t is the beginning of period stock price, D_t is the dividend paid during period t, R_t is the return on the stock from period t to $t+1$ and E_t is the expectation at time t. Note that we don't explicitly include an unobserved variable, leaving its impact to enter the bubble term. Rearranging and taking logarithms yields

$$r_t = E_t p_{t+1} - p_t + \ln(1 + e^{E_t(d_t - p_{t+1})})$$

where lower case letters denote logarithms of upper case letters and $r_t = \ln(1 + R_t)$. Taking a first-order Taylor expansion around $x_t = E_t(d_t - p_{t+1})$ yields

$$r_t = k + (1-\psi)E_t d_t + \psi E_t p_{t+1} - p_t$$

where $\psi = \frac{1}{1+e^{d-p}}$ and $k = -\ln\psi + (1-\psi)\ln(\frac{1}{\psi} - 1)$. Rearranging and iterating forward results in

$$p_t = \frac{k}{1-\psi} + (1-\psi)\, E_t \sum_{i=0}^{\infty} \psi^i d_{t+i} - E_t \sum_{i=0}^{\infty} \psi^i r_{t+i} + b_t \tag{3.3}$$

with

$$E_t(b_{t+i}) = \left(\frac{1}{\psi}\right)^i b_t \tag{3.4}$$

First difference these equations to get

$$\Delta p_t = (1-\psi) \sum_{i=0}^{\infty} \psi^i \left[E_t d_{t+i} - E_{t-1} d_{t+i-1}\right] - \sum_{i=0}^{\infty} \psi^i \left[E_t r_{t+i} - E_{t-1} r_{t+i-1}\right] + \Delta b_t \tag{3.5}$$

$$\Delta b_t = \left(\frac{1}{\psi}\right) \Delta b_{t-1} \tag{3.6}$$

In case of a constant real interest rate r (3.3) reduces to

$$p_t = \frac{k-r}{1-\psi} + (1-\psi)\, E_t \sum_{i=0}^{\infty} \psi^i d_{t+i} + b_t$$

and (3.5) to

$$\Delta p_t = (1-\psi)\sum_{i=0}^{\infty}\psi^i\left[E_t d_{t+i} - E_{t-1}d_{t+i-1}\right] + \Delta b_t \tag{3.7}$$

In the following we sketch the principles of the state-space approach and its estimation[14]. We then apply it to the present value model. The state-space model consists of the system of equations:

$$\underset{(n\times 1)}{y_t} = \underset{(n\times k)(k\times 1)}{Ax_t} + \underset{(n\times r)(r\times 1)}{Hs_t} + \underset{(n\times 1)}{w_t} \tag{3.8}$$

$$\underset{(r\times 1)}{s_{t+1}} = \underset{(r\times r)(r\times 1)}{Fs_t} + \underset{(r\times 1)}{v_{t+1}} \tag{3.9}$$

(3.8) is the measurement or observation equation, which describes the relation between observed and unobserved variables, where y_t, and x_t are vectors of observed variables and s_t is a vector of unobserved variables and w_t is an error term. (3.9) is the state equation, which describes the dynamics of the unobserved variables vector s_t, where v_{t+1} is an error term. A, H and F are coefficient matrices that have to be estimated from the data. Maintained assumptions are

$$E\left(v_t v_\tau'\right) = \begin{cases} Q \text{ for } t=\tau \\ 0 \text{ otherwise} \end{cases}$$

$$E\left(w_t w_\tau'\right) = \begin{cases} R \text{ for } t=\tau \\ 0 \text{ otherwise} \end{cases}$$

$$E\left(v_t w_\tau'\right) = 0 \text{ for all } t \text{ and } \tau$$

$$E\left(v_t s_1'\right) = 0 \text{ for all } t$$

$$E\left(w_t s_1'\right) = 0 \text{ for all } t$$

The objective of the Kalman filter is to find linear least squares forecasts of the state vector s_t. Suppose the coefficient matrices were known, then

$$\begin{aligned} \hat{s}_{t+1|t} &= \hat{E}\left(s_{t+1} \mid y_t, y_{t-1}, ..., y_1, x_t, x_{t-1}, ..., x_1\right) \\ &= F\hat{E}\left(s_t \mid y_t, y_{t-1}, ..., y_1, x_t, x_{t-1}, ..., x_1\right) \\ &= F\hat{s}_{t|t} \end{aligned} \tag{3.10}$$

[14] For a thorough discussion refer to Hamilton's (1994) textbook.

$\hat{s}_{t|t}$ is the forecast $\hat{s}_{t|t-1}$ updated by new information in y_t.

$$\begin{aligned}\hat{s}_{t|t} &= \hat{s}_{t|t-1} + \left\{E\left[\left(s_t - \hat{s}_{t|t-1}\right)\left(y_t - \hat{y}_{t|t-1}\right)'\right]\right\} \\ &\quad \times \left\{E\left[\left(y_t - \hat{y}_{t|t-1}\right)\left(y_t - \hat{y}_{t|t-1}\right)'\right]\right\}^{-1} \times \left(y_t - \hat{y}_{t|t-1}\right)\end{aligned}$$

Updating the forecast is done by adding to it the unanticipated part of the new piece of information $y_t - \hat{y}_{t|t-1}$ weighted by a matrix, which could be interpreted as the correlation of the state and measurement equation forecast error. The larger the correlation the more weighs the arrival of new information. To compute the updated forecast one needs a forecast of y_t.

$$\begin{aligned}\hat{y}_{t|t-1} &= \hat{E}\left(y_t \mid y_{t-1}, y_{t-2}, ..., y_1, x_t, x_{t-1}, ..., x_1\right) \\ &= Ax_t + H\hat{s}_{t|t-1}\end{aligned} \tag{3.11}$$

The Kalman filter is started by setting starting values for $s_{1|0}$ and an associated mean squared error

$$P_{1|0} = E\left\{\left[s_1 - E\left(s_1\right)\right]\left[s_1 - E\left(s_1\right)\right]'\right\}$$

For stationary processes $s_{1|0}$ is set to the unconditial mean of the process and the initial mean squared error can be computed from the matrices F and Q. For non-stationary processes $s_{1|0}$ is set to some best guess and $P_{1|0}$ arbitrarily high to reflect the uncertainty about $s_{1|0}$. Iterate over (3.10) to (3.11) to find the series $\left\{\hat{s}_{t|t-1}\right\}_{t=1}^{T}$ and $\left\{P_{t|t-1}\right\}_{t=1}^{T}$.

The system (3.8) and (3.9) is estimated by maximising the loglikelihood function

$$\sum_{t=1}^{T} \log f\left(y_t \mid y_{t-1}, ..., y_1, x_t, x_{t-1}, ..., x_1\right)$$

To do so, set the matrices A, H, F, Q, R to some initial values, find the series $\left\{\hat{s}_{t|t-1}\right\}_{t=1}^{T}$ and $\left\{P_{t|t-1}\right\}_{t=1}^{T}$ by the Kalman filter and calculate the value of the loglikelihood function. Numerical optimisation procedures can be employed to maximise the loglikelihood function.

We follow Wu (1997) and estimate a state-space system with the stock price equation (3.7) as measurement equation and the unobserved bubble process (3.4) as state equation. The difference to Wu (1997) at this point is that while he uses data up to 1992 we extend the sample range up to 2005. Also we estimate a version of the model with a time-varying interest rate, while Wu (1997) assumes a constant real interest rate throughout. The stock price equation is first-differenced because the stock price and dividends series are non-stationary.

Indeed, log dividends are found to follow an ARIMA $(h, 1, 0)$ process[15]

$$\Delta d_t = \mu + \sum_{i=1}^{h} \varphi_i \Delta d_{t-i} + \delta_t \tag{3.12}$$

where the lag length h is determined by the data. (3.12) can be written in the companion form

$$z_t = u + Bz_{t-1} + \nu_t \tag{3.13}$$

where $z_t = (\Delta d_t, \Delta d_{t-1}, ..., \Delta d_{t-h+1})'$, $u = (\mu, 0, ..., 0)'$ and $\nu_t = (\delta_t, 0, ..., 0)'$ are h-vectors and

$$B = \begin{pmatrix} \varphi_1 & \varphi_2 & \cdots & \varphi_{h-1} & \varphi_h \\ 1 & 0 & ... & 0 & 0 \\ 0 & 1 & ... & 0 & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & 0 & ... & 1 & 0 \end{pmatrix}$$

is a $h \times h$-matrix. According to Wu (1997) and Campbell and Shiller (1987) the solution to (3.7) can then be obtained using (3.13) in

$$\Delta p_t = \Delta d_t + M \Delta z_t + \Delta b_t$$

where $M = gB(I-B)^{-1} \left[I - (1-\psi)(I-\psi B)^{-1}\right]$ and $g = (1, 0, ..., 0)$ are h-row vectors and I the $h \times h$-identity matrix.

3.6.2 Empirical results

In our case to determine the optimal lag length h of the first differences of dividends we used the AIC and SBC criteria as well as an LR-ratio test. The resulting optimal lag length is $h = 6$ as reported in table 3.10. Together with (3.6) this implies the following state-space model

$$\Delta p_t = \sum_{i=0}^{6} \alpha_i \Delta d_{t-i} + \Delta b_t \tag{3.14}$$

$$\Delta b_t = \gamma \Delta b_{t-1} + \zeta_t \tag{3.15}$$

Table 3.4 reports the estimation results of the parameters α_i and σ_ζ together with their standard errors and significance levels. Initial values for the coefficients were taken from a simple OLS regression of the measurement equation.

Clearly all coefficients except those on the lag of the first difference of the bubble term are insignificant. The coefficient on the lagged difference of the bubble term is significant. Also the estimated standard deviation of the state

[15] For all variables the time-series processes have been estimated.

	coefficient	std. error	prob.
Δd_t	0.1865	0.3647	0.6090
Δd_{t-1}	0.1783	0.2817	0.5268
Δd_{t-2}	−0.2329	0.3073	0.4486
Δd_{t-3}	−0.1618	0.3077	0.5990
Δd_{t-4}	−0.1034	0.3253	0.7505
Δd_{t-5}	0.3613	0.2815	0.1993
Δd_{t-6}	0.2268	0.3183	0.4760
Δb_{t-1}	0.3634	0.0701	0.0000
σ_ζ	0.0553	0.0020	0.0000

Table 3.4: Estimation results of coefficients in stock price and bubble equation, constant real interest rate

equation is significant. Figure 3.2 shows the estimated bubble in levels with the corresponding 95%-confidence bands[16]. A unit root test on the estimated

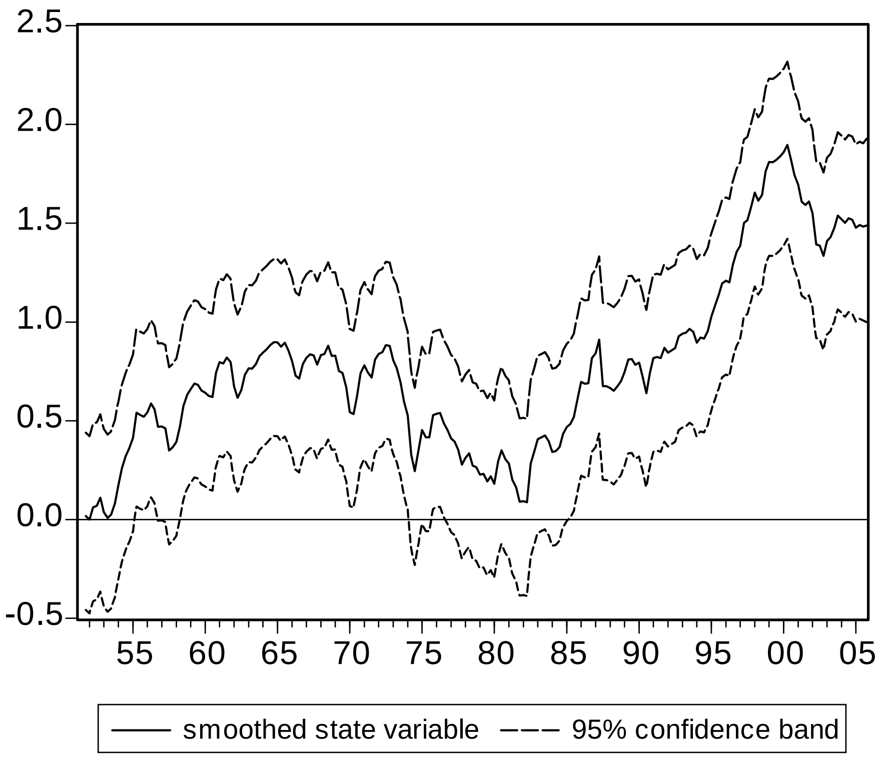

Figure 3.2: Smoothed estimate of state variable in levels with 95%-confidence bands, constant real interest rate

state variable is not easily performed since it would be based on estimated data,

[16]We report results based on smoothed estimates of the state equation in levels, which means that in (3.10) the Kalman filter uses all available observations $t = 1, ..., T$ to estimate the unobserved state

$$\hat{s}_{t+1|T} = \hat{E}\left(s_{t+1} \mid y_T, y_{T-1}, ..., y_t, ..., y_1, x_T, x_{T-1}, ..., x_t, ..., x_1\right) \tag{3.16}$$

such that the critical value normally applied to unit root tests might not be valid. However, from visual inspection the smoothed state variable appears to be rather in line with the notion of periodically collapsing bubbles, which would make the bubble term appear integrated of order one or zero, rather than with explosive behaviour. There are clearly periods in which the estimated bubble term is positive and significant, e.g. during most of the late 1950s through the early 1970s and especially in the late 1990s. Table 3.5 reports the periods during which the estimated state variable series is significantly different from zero.

Period	Sign of state variable
1955 Q2 - 1956 Q3	+
1958 Q3 - 1974 Q1	+
1975 Q4 - 1976 Q3	+
1985 Q2 - 2005 Q4	+

Table 3.5: Significant bubble episodes, constant real interest rate

Getting a precise estimate for the size of the bubble requires an assumption about the size of the bubble at the starting date. Conservatively we have set this starting value to zero in the estimation[17]. Even with this assumption the bubble is significant during plausible periods. However, even without the exact size of the bubble we can test determinants that might have influenced the bubble. Obviously the assumption of a constant real interest rate might be quite restrictive and responsible for the high stock price index. Therefore we next estimate the same model as above including a time-varying interest rate. In particular, we use the following specification.

$$\Delta p_t = \sum_{i=0}^{6} \alpha_i \Delta d_{t-i} + \sum_{j=0}^{7} \beta_j \Delta r_{t-j} + \Delta b_t \tag{3.17}$$

$$\Delta b_t = \gamma \Delta b_{t-1} + \zeta_t \tag{3.18}$$

The lag length of the real interest rate series is taken from table 3.10 as before and is also based on the AIC, SBC and LR-ratio. Table 3.6 reports the results. The coefficients on the contemporaneous values and most of the lags of the first differences of dividends and the real interest rate are insignificant. However, only those on the fifth lag of dividends and the third lag of the real interest rate are significant. Also the coefficient on the lagged differenced state variable and its standard deviation are significant again. Figure 3.3 plots the smoothed estimate of the level of the state variable. The starting value of the state variable was set to zero as before. Table 3.7 reports the periods during which the estimated

[17] Theoretically, a rational bubble can only start at the first day of trading (Diba and Grossman, 1988b). Thus, we also ran the estimation setting the starting value of the state value to a very small number with the same results.

	coefficient	std. error	prob.
Δd_t	0.2660	0.3894	0.4946
Δd_{t-1}	0.2553	0.3474	0.4625
Δd_{t-2}	-0.3464	0.3319	0.2967
Δd_{t-3}	-0.4516	0.3573	0.2062
Δd_{t-4}	-0.0444	0.3173	0.8887
Δd_{t-5}	0.5078	0.2853	0.0751
Δd_{t-6}	0.2910	0.3848	0.4495
Δr_t	0.9835	0.6881	0.1529
Δr_{t-1}	-0.4535	0.5459	0.4062
Δr_{t-2}	-0.2192	0.5635	0.6973
Δr_{t-3}	0.7661	0.4373	0.0798
Δr_{t-4}	-0.8610	0.6643	0.1949
Δr_{t-5}	-0.0009	0.5857	0.9988
Δr_{t-6}	-0.3782	0.6202	0.5420
Δr_{t-7}	0.8848	0.7041	0.2089
Δb_{t-1}	0.3372	0.0772	0.0000
σ_ζ	0.0537	0.0024	0.0000

Table 3.6: Estimation results of coefficients in stock price and bubble equation, time-varying real interest rate

state variable series is significantly different from zero.

Period	Sign of state variable
1964 Q3 - 1965 Q4	+
1968 Q2 - 1969 Q1	+
1972 Q3 - 1972 Q4	+
1987 Q2	+
1991 Q1	+
1991 Q4 - 2005 Q4	+

Table 3.7: Significant bubble episodes, time-varying real interest rate

Including a time-varying interest rate eliminates the bubble in the 1950s, in most of the 1960s and 70s and some of the bubble in the early 1990s. Still, the level of the S&P 500 just before the Black Monday stock market crash is found to contain a bubble. From this one can conclude that a time-varying interest rate is non-negligible in explaining real stock prices.

Another issue that arises when estimating the state variable process is the construction of the confidence intervals. Generally, when making a forecast based on estimated processes there are two sources of uncertainty, which should be reflected in the standard errors: the forecast uncertainty and the estimation uncertainty. Lütkepohl (2004) argues that in large samples the estimation uncertainty becomes negligible. In the present case, for $T = 200$ and $T = 218$, respectively, this means that one can use the residuals from the estimated state equation to compute the confidence bands. However, Lütkepohl (2005) also de-

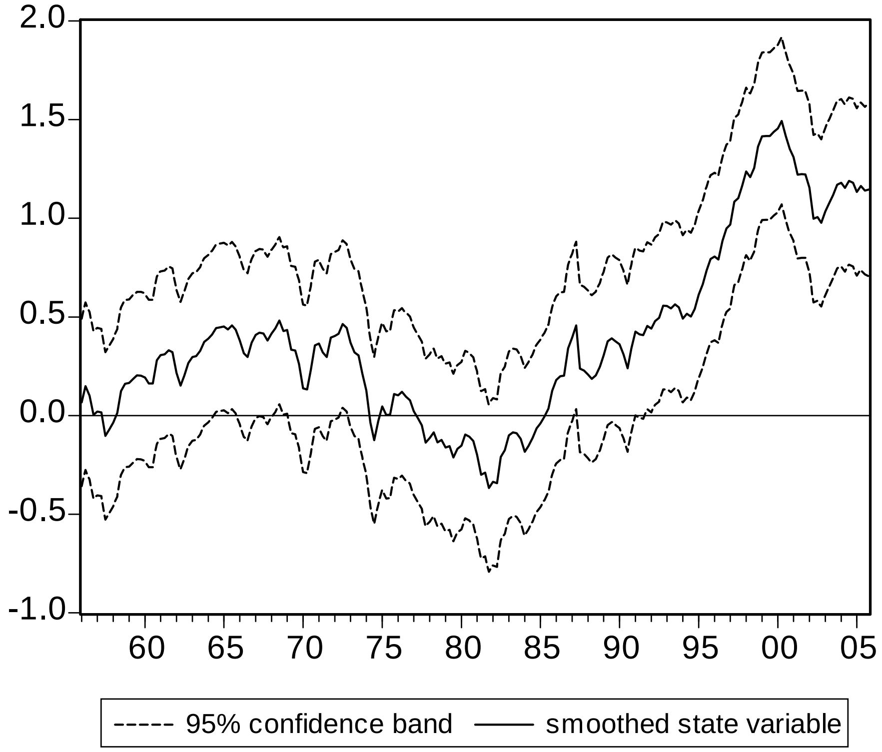

Figure 3.3: Smoothed estimate of state variable in levels with 95%-confidence bands, time-varying real interest rate

rives an approximation of the estimation uncertainty in small samples. As a robustness check, we derive confidence intervals for the state variable including an approximate estimation error in the appendix.

To sum up, the estimation of an unobserved variable which influences stock prices over and above dividends yields a statistically significant coefficient on the lagged differenced bubble component in the present value model. In addition the state-space framework delivers estimates for the process of the level of the unobserved variable over the sample period. It also provides information about the periods during which the unobserved variable significantly deviates from zero. Consequently we can show that there is a substantial deviation of the unobserved variable from zero during the suspected bubble period in the late 1990s. In what follows we use the estimated process for the unobserved variable to test a number of variables for their explanatory power. These variables are proxies for moral hazard behaviour of investors.

3.7 Indicators of moral hazard behaviour of investors

The main problem with analysing the empirical content of the Greenspan-put hypothesis is that moral hazard behaviour of investors is not observed. We choose the second best option and rely on theory to find indicators. In particular, we construct measures of moral hazard behaviour based on the models by Illing (2001) and Miller, Weller and Zhang (2002).

3.7.1 The probability of a stock market crash

Illing (2001) sets up a model in which the central bank wants to avoid disruption of the financial sector because this leads to a loss of informational capital and the inefficient liquidation of solvent firms. There is one safe old economy sector and one risky new economy sector. In case of an aggregate shock to the new economy sector a share λ of failing firms can be restructured with continuation value C, the share $(1-\lambda)$ is liquidated early at the liquidation value L. Due to their informational capital it is only the relationship lender bank that knows which firms are worth being restructured and continued. An aggregate shock might lead to financial disruption if it triggers a bank run. Then aggregate losses would equal $\lambda(C-L)$. A bank run occurs if aggregate debt exposure is larger than what can be recovered in case of an aggregate shock. To avoid any risk of a bank run and the subsequent financial disruption, the central bank must inject enough liquidity to reduce the real value of debt. The value of real debt must equal the liquidation value of firms. The reduced real debt burden is a capital gain to the restructured firms in the new economy sector because their continuation value C is now larger than their real debt burden, which equals L. If rational investors anticipate these capital gains they include it in the valuation of the new economy firms driving up their asset price over the fundamental value by the amount of the expected capital gain. The difference equals the asset price bubble B_t.

$$B_t = \xi_t \lambda_t (C_t - L_t)$$

where ξ_t is the probability of an aggregate shock. The asset price bubble depends positively on the probability of an aggregate shock, the share of restructured firms and the efficiency loss avoided. We use the probability of an aggregated shock to test for moral hazard among investors. In our case ξ_t is the probability of a stock market crash. To measure it we rely on a survey among institutional investors in the U.S. by Robert Shiller. In his Crash Confidence Index he reports the percentage of respondents who think that the probability of a stock market crash in the following six months is less than 10%[18].

3.7.2 A minimum level of dividends

A second measure has been constructed from a theoretical model by Miller, Weller and Zhang (2002). They set up a continuous time model of stock prices where dividends follow a Brownian motion. The source of stock price movements over and above dividend growth are jumps in the dividend process, which are interpreted as "periodic large adverse movements which we shall term 'crises'" (Miller, Weller and Zhang, 2002). These jumps raise the risk-premium and thus lower the stock price. However, if the central eliminates the downward jumps by providing sufficient liquidity when a fall in dividends is likely to occur, the risk-premium falls and the stock price rises. Under the assumption that investors believe that the central bank will prevent dividends from falling sharply and under various scenarios of parameter values for the real interest rate, the risk-premium, the dividend growth rate etc. Miller, Weller and Zhang (2002) can generate quite large stock price overvaluations of up to 204%. They assume that investors believe that the central bank will stabilise the market at some fraction κ of the latest stock market peak $\bar{P}$. More precisely, dividends are prevented from falling below the minimum level D_b which is given by

$$P\left(D_b\right) = \kappa \bar{P}$$

The current dividend level can be expressed as a multiple of the minimum level, $D = m\ D_b$. The stock price bubble B then depends negatively on the ratio m because the put option given by the central bank is worth more the closer actual dividends get to the exercise value D_b.

$$B = B(\underset{-}{m})$$

As a measure of m we use the ratio of actual dividends to their level in the period after the stock price crash 1987 Q4[19]. This takes account of the argument that the Fed created the expecetation of a bail-out guarantee by its reaction to the

[18] Survey data on investors' stock market confidence has been collected since 1984 by Robert Shiller within the Investor Behavior Project at the Yale International Center for Finance. The questionnaire has been sent to a number of U.S. investors who have been sampled from the investment managers section of the Money Market Directory of Pension Funds and Their Investment Managers. The average sample size in each survey round has been about a hundred. From the data several indices relating to different aspects of stock market confidence are constructed, among them the percentage of respondents who think that the probability of a stock market crash in the following six months is less than 10%. More on the methodology of Shiller's stock market crash confidence index can be found on *http://icf.som.yale.edu/confidence.index/CrashIndex.shtml.*

[19] $\kappa = 0.79$ as determined by the data.

Black Monday stock market crash. In addition, since the same is said to be true of the LTCM crisis in 1998, we construct an alternative measure of m by letting the stock price in 1998 Q2 set a new peak, at 79% of which the Fed was supposed to intervene.

3.7.3 The degree of debt exposure

In addition we use the argument that a central bank's incentive to intervene in a stock market crisis rises with the degree of leverage. Illing (2001) argues that with higher leverage, i.e. the debt-gdp-ratio, the risk of a bank run and of a financial crisis rises, which in turn should lead to the build up of a bubble. Borio and Lowe (2002) present evidence in an explorative study that the deviation of the debt-gdp-ratio from its trend and the real credit growth are reasonably good predictors of financial crises. We are aware that this measure is rather weak because there might be a simultaneity problem. High credit growth might be caused by high asset price growth and vice versa. We try to avoid this problem by including only lagged values of credit growth and its deviation from trend in the tests.

3.7.4 Empirical results

There are five different indicators of moral hazard behaviour derived from the theoretical models discussed above: The probability of a stock market crash, two versions of the ratio of current dividends to a minimum level as perceived to be guaranteed by the central bank, the deviation of the debt-gdp-ratio from trend and the growth of real debt outstanding. The indicators were tested in two ways. First, we checked whether each indicator, appropriately differenced and lagged, had a significant impact on the residuals from the state equation in levels.

$$\hat{\eta}_t = \sum_{i=0}^{n} \theta_i x_{t-i} + \omega_t \tag{3.19}$$

where $\hat{\eta}$ are the fitted residuals from the state equation in levels, x_t is one of the different moral hazard indicators and ω is an error term. The lag length n is determined by the data. Unless an indicator follows an $AR(1)$ process itself any significant influence on the bubble should show up in this test. Second, we included each indicator in turn in the measurement equation.

$$\Delta p_t = \sum_{i=0}^{6} \alpha_i \Delta d_{t-i} + \sum_{j=0}^{7} \beta_j \Delta r_{t-j} + \sum_{i=0}^{n} \theta_i x_{t-i} + \Delta b_t \tag{3.20}$$

If moral hazard behaviour had any impact on stock prices the coefficients θ_i of the indicator should be significant. One could view this specification as allowing for the part of a time-varying risk-premium, which is assumed to be influenced by the degree of moral hazard. A specification analysis of the time-series of the indicators on the basis of unit root tests, the AIC criterion and tests for autocorrelation was performed. We used the logs of the first three indicators because of the logarithmic formulation of our baseline estimation. The last two indicators are already in growth rates and deviation from trend, respectively. All indicators start in 1987 Q4 the period after the Black Monday stock market crash.

To make the different approaches in this paper consistent we based them all on the same dataset. It is well known that unit root tests are quite sensitive to the sample length, which is why we opted for the maximum length available. However, these data are only available as monthly averages. As Working (1960) has pointed out the use of averages might result in autocorrelation of the series in first differences when in fact the series in levels follows a random walk. Unfortunately this proposition can't be tested on the data we used because non-averaged data is not available for the maximum sample length. However, the results based on averaged data yields plausible estimates for potential deviations of stock prices from their fundamentals as specified by the present value model. This is further supported by the results from the unit root/cointegration analysis, which cannot reject the presence of an unobservable variable that follows a stochastic trend. Furthermore, the tests for the impact of the various moral hazard indicators in the augmented measurement equation serve as a cross-check for the results derived from the state equation. Thus, even if the autocorrelation in the state equation was induced by the use of averaged data, the indicators should still be significant in the measurement equation. A non-significant coefficient in the state equation could result because the filter might not detect any unobservable variable that doesn't exactly follow an AR(1) process, which the various indicators don't. After all the analysis is not intended as a test for the existence of bubbles per se but rather as a test for the impact of measures of moral hazard on stock prices.

Table 3.8 reports the results for the first test. The only indicator with a significance level of below 10% is the contemporaneous ratio of current dividends to a minimum level in the test for misspecification of the state equation. Note however, that the coefficient is positive, contrary to what Miller, Weller and Zhang's (2002) model predicts. The results across all other indicators clearly reject the Greenspan-put hypothesis. None of the other indicators has a significant impact on the residuals from the state equation. Furthermore, only the first lag of the growth rate of debt-to-gdp in the augmented measurement equation is marginally significant on the 10%-level with the correct sign. None of the other indicators has any significant impact on the log-differenced stock price in the

Test for misspecification of state equation (3.19)

$$\hat{\eta}_t = \sum_{i=0}^{n} \theta_i x_{t-i} + \omega_t$$

time-varying real interest rate

indicator x_{t-i}	no. lags	lag i	coefficient	std. error	prob.
$\Delta \ln \xi_{t-i}$	0	0	−0.0068	0.0147	0.6427
$\Delta \ln m_{t-i}^{1987}$	4	0	0.8815	0.5089	0.0881
		1	−0.1229	0.5484	0.8234
		2	0.5438	0.5315	0.3101
		3	0.1305	0.5411	0.8102
		4	−0.6353	0.5113	0.2186
$\Delta \ln m_{t-i}^{1998}$	2	0	0.0485	0.2043	0.8130
		1	0.1477	0.2030	0.4693
		2	0.2038	0.2033	0.3198
$\frac{Debt}{GDP}$ gap$_{t-i}$	3	1	−0.4430	0.8495	0.6026
		2	−0.5878	0.9999	0.5573
		3	0.9538	0.8265	0.2499
$\frac{Debt}{GDP}$ growth$_{t-i}$	6	1	−0.7731	1.5036	0.6077
		2	1.9447	1.6951	0.2527
		3	0.7100	1.7621	0.6875
		4	0.4953	1.7387	0.7761
		5	−0.8890	1.6286	0.5858
		6	−1.0082	1.4221	0.4792

Table 3.8: Impact of moral hazard indicators on residuals from state equation in levels

measurement equation as table 3.9 shows.

Altogether this suggests at a minimum that the predictions of the models by Illing (2001) and Miller, Weller and Zhang (2002) are not confirmed by the data. Under the assumption that the indicators are valid measures of moral hazard behaviour of investors, the results also indicate a clear rejection of the Greenspan-put hypothesis.

3.8 Conclusion

The research objective of this paper is to investigate the empirical content of the Greenspan-put hypothesis. It claims that investors believed in an implicit bail-out guarantee by the Fed should the stock market crash. The Fed is believed to inject liquidity into the market in a stock market crash as it has done a number of times in the past and as Greenspan (2004) claims to have done. This is supposed to have contributed to the build-up of the bubble in the late 1990s. Using the present value model of stock prices we have identified an unobserved variable which is integrated of order one as a determinant of stock prices from 1950 to 2005. Since periodically collapsing bubbles might appear integrated of order one

Augmented stock price equation (3.20)

$$\Delta p_t = \sum_{i=0}^{6} \alpha_i \Delta d_{t-i} + \sum_{j=0}^{7} \beta_j \Delta r_{t-j} + \sum_{i=0}^{n} \theta_i x_{t-i} + \Delta b_t$$

indicator x_{t-i}	no. lags	lag i	coefficient	std. error	prob.
$\Delta \ln \xi_{t-i}$	0	0	0.0068	0.0409	0.8674
$\Delta \ln m_{t-i}^{1987}$	4	0	−0.2324	0.7461	0.7554
		1	0.8337	0.9330	0.3715
		2	1.2449	0.9990	0.2127
		3	−0.4265	0.8995	0.6354
		4	0.6810	0.7932	0.3906
$\Delta \ln m_{t-i}^{1998}$	2	0	0.0134	0.3217	0.9667
		1	−0.1351	0.1847	0.4644
		2	0.1152	0.1889	0.5420
$\frac{Debt}{GDP}$ gap_{t-i}	3	1	−0.7011	0.9852	0.4767
		2	0.2336	1.2268	0.8490
		3	−0.0216	1.0972	0.9843
$\frac{Debt}{GDP}$ growth_{t-i}	6	1	2.8048	1.7313	0.1052
		2	−1.0509	2.2883	0.6461
		3	−1.4858	1.8944	0.4329
		4	1.1092	2.4762	0.6542
		5	0.3422	2.0680	0.8686
		6	−0.6819	1.8624	0.7143

Table 3.9: Impact of moral hazard indicators on stock prices in measurement equation

or zero (Evans, 1991) we have estimated a state-space model and have identified periods of significant estimates of the unobserved time-series component which we take as a bubble. One is during the 1960s and the early 1970s and the other one in the late 1990s. This allows to test for various measures of moral hazard behaviour of investors. These measures are constructed on the basis of theoretical models because moral hazard itself is not observable. Our results show that none of the moral hazard indicators has any explanatory power in either the bubble process itself or the stock price equation. The bubble in the late 1990s can't be explained by measures of moral hazard. However, we find that a large part of the bubble can be explained by time variations in the real interest rate.

One criticism of our approach might be that the measures of moral hazard are false. However, the measures are based on theory and thus, at a minimum, we are able to reject the predictions of the models by Illing (2001) and Miller, Weller and Zhang (2002). Moreover, there is the so-called Peso problem: Rational expectations of some event, like a future tax cut, might have pushed up stock prices, even though the expected event didn't materialize later on. Since these expectations are not observed and it is very hard to find a measure based on theory, we can't control for them. Finally, it may be possible that moral hazard considerations weigh more in more narrow stock market indices than the S&P 500. Applying the approach laid out in this paper to other indices might yield further insights. All in all we are unable to confirm the hypothesis that there existed the wide-spread belief in a Greenspan put option as tested on the S&P 500 stock index. This suggests either that U.S. investors truly didn't believe in an implicit bail-out guarantee after having observed the Fed's rescue operations in the past, or that currently existing models don't fully capture the moral hazard element and further research in that area is needed.

References

Allen, F. and D. Gale (2000): Bubbles and crises. *The Economic Journal*, Vol. 110, N^o 460, 236-255.

Bernanke, B. and M. Gertler (2001): Should central banks respond to movements in asset prices? *The American Economic Review*, Vol. 91, N^o 2, 253-257.

Bhargava, A. (1986): On the theory of testing for unit roots in observed time series. *The Review of Economic Studies*, Vol. 53, N^o 3, 369-384.

Borio, C. and P. Lowe (2002): Asset prices, financial and monetary stability: exploring the nexus. BIS working papers N^o 114.

Borio, C. and P. Lowe (2003): Imbalances or "bubbles?" Implications for monetary and financial stability, in: Hunter, W., G. Kaufman and M. Pomerleano: *Asset price bubbles. The implications for monetary, regulatory and international policies*. MIT Press, Cambridge and London.

Campbell, J. and R. Shiller (1987): Cointegration tests of present value models. *The Journal of Political Economy*, Vol. 95, N^o 5, 1062-1088.

Cao, J. and G. Illing (2007): Liquidity shortages and monetary policy. Munich Economics Discussion Paper No. 2007-25.

Cecchetti, S., H. Genberg, J. Lipsky and S. Wadhwani (2000): Asset prices and monetary policy. Geneva Reports on the World Economy 2. Oxford.

Cogley, T. (1999): Should the Fed take deliberate steps to deflate asset price bubbles? *Federal Reserve Bank of San Francisco Economic Review,* N^o 1, 42-52.

Diba, B. and H. Grossman (1987): On the inception of bubbles. *The Quarterly Journal of Economics*, Vol. 102, N^o 3, 697-700.

Diba, B. and H. Grossman (1988a): Explosive rational bubbles in stock prices? *The American Economic Review*, Vol. 78, N^o 3, 520-530.

Diba, B. and H. Grossman (1988b): The theory of rational bubbles in stock prices. *The Economic Journal*, Vol 98, N^o 392, 746-754.

Driffil, J. and M. Sola (1998): Intrinsic bubbles and regime-switching. *Journal of Monetary Economics*, Vol. 42, 357-373.

Evans, G. (1991): Pitfalls in testing for explosive bubbles in asset prices. *The American Economic Review*, Vol. 81, N^o 4, 922-930.

Filardo, A. (2004): Monetary policy and asset price bubbles: calibrating the monetary policy trade-offs. BIS working paper N^o 155.

Financial Times (2001): Lex - The Greenspan put. 4 January 2001.

Froot, K. and M. Obstfeld (1991): Intrinsic bubbles: the case of stock prices. *The American Economic Review*, Vol. 81, N^o 5, 1189-1214.

Greenspan, A. (1999): General Discussion: monetary policy and asset price volatility, in: New Challenges for Monetary Policy. Symposium sponsored by the Federal Reserve Bank of Kansas City Jackson Hole, Wyoming.

Greenspan, A. (2004): Risk and uncertainty in monetary policy. Remarks at the meeting of the American Economic Association, San Diego, California, January 3, 2004.

Gurkaynak, R. (2005): Econometric tests of asset price bubbles: taking stock. Finance and Economics Discussion Series. Federal Reserve Board, Washington, D.C.

Hall, S., Z. Psaradakis and F. Sola (1999): Detecting periodically collapsing bubbles: a markov-switching unit root test. Journal of Applied Econometrics, Vol. 14, N^o 2, 143-154.

Hamilton, J. (1994): *Time series analysis*. Princeton University Press.

Illing, G. (2001): Financial fragility, bubbles and monetary policy. CESifo working paper N^o 449.

Lettau, M., S. Ludvigson and J. Wachter (2006): The declining equity premium: what role does macroeconomic risk play? *Review of Financial Studies*, forthcoming.

LeRoy, S. and R. Porter (1981): The present-value relation: tests based on implied variance bounds. *Econometrica*, Vol. 49, 555-574.

Lütkepohl, H. (2004): Univariate time series analysis. In: Lütkepohl, H. and M. Krätzig (ed.): *Applied time series econometrics*. Cambridge, 1st edition.

Lütkepohl, H. (2005): *New introduction to multiple time series analysis.* Springer, Germany.

Miller, M., P. Weller and L. Zhang (2002): Moral hazard and the US stock market: analysing the 'Greenspan put'. *The Economic Journal*, Vol. 112, No. 478, C171-C186.

Mishkin, F. and E. White (2003): U.S. stock market crashes and their aftermath: implications for monetary policy, in: Hunter, W., G. Kaufman and M. Pomerleano: *Asset price bubbles. The implications for monetary, regulatory and international policies.* MIT Press, Cambridge and London.

Mussa, M. (2003): Remarks on achieving monetary and financial stability, in: *Monetary stability, financial stability and the business cylce: five views.* BIS papers N^o 18

Pastor, L. and P. Veronesi (2006): Was there a Nasdaq bubble in the late 1990s? *Journal of Financial Economics*, Vol. 81, 61-100.

Psaradakis, Z., M. Sola and F. Spagnolo (2001): A simple procedure for detecting periodically collapsing rational bubbles. *Economics Letters*, Vol. 72, 317-323.

Psaradakis, Z., M. Sola and F. Spagnolo (2004): On markov error-correction models, with an application to stock prices and dividends. *Journal of Applied Econometrics*, Vol. 19, 69-88.

Sarno, L. and M. Taylor (1999): Moral hazard, asset price bubbles, capital flows, and the East Asian crisis: the first tests. *Journal of International Money and Finance*, Vol. 18, 637-657.

Sauer, S. (2007): Liquidity risk and monetary policy. Munich Economics Discussion Paper No. 2007-27.

Shiller, R. (1981): Do stock prices move too much to be justified by subsequent changes in dividends? *The American Economic Review*, Vol. 71, N^o 3, 421-436.

Shiller, R. (2001): *Irrational Exuberance*. Broadway Books. New York.

Taylor, M. and D. Peel (1998): Periodically collapsing stock price bubbles: a robust test. *Economics Letters*, Vol. 61, 221-228.

The Economist (1999): Leader *and* Survey: The world economy. 23 September 1999.

The Economist (2006): Special report Alan Greenspan. 14 January 2006.

West, K. (1987): A specification test for speculative bubbles. *The Quarterly Journal of Economics*, Vol. 102, N^o 3, 553-580.

Working, H. (1960): Note on the correlation of first differences of averages in a random chain. *Econometrica*, Vol. 28, No. 4, 916-918.

Wu, Y. (1995): Are there rational bubbles in foreign exchange markets? Evidence from an alternative test. *Journal of International Money and Finance*, Vol. 14, N^o 1, 27-46.

Wu, Y. (1997): Rational bubbles in the stock market: accounting for the U.S. stock-price volatility. *Economic Inquiry*, Vol. 35, 309-319.

Appendix 3.A Unit root and cointegration tests with a time-varying interest rate

ADF regression in log-levels

$$\Delta y_t = \mu + \gamma t + \rho y_{t-1} + \sum_{i=1}^{n} \beta_i \Delta y_{t-i} + \epsilon_t$$

$H_o : \rho = 0$, unit root in y_t

$T = 224$ for p_t and d_t, $T = 207$ for r_t

y_t	p_t	Δp_t	d_t	Δd_t	r_t	Δr_t	$\binom{p_t}{d_t}$
		$\gamma = 0$		$\gamma = 0$	$\gamma = 0$	$\gamma = 0$	$\gamma = 0$
n^o of lags	2	1	7	6	7	7	2
t-statistic on ρ	-0.986	-13.717^*	-1.957	-8.871^*	-0.718	-9.897^*	-2.652
5% critical value	-3.423	-2.876	-3.423	-2.876	-2.876	-2.876	-3.380

Table 3.10: ADF unit root and cointegration tests on the natural logarithm of the real stock price, dividends and the real interest rate

Table 3.10 presents the results for unit root tests on the present value model with a time-varying interest rate using the S&P 500 dividend index series. The fact that the real interest rate seems integrated of order 1 in table 3.10 is probably due to a structural break at the beginning of the 1980s when inflation fell sharply due to Paul Volcker's tight monetary policy and the real interest rate soared. For the purpose of our analysis we disregard formally accounting for a structural break in the unit root tests because our conclusions don't depend on it. However, a Chow breakpoint and Chow forecast test rejects the null of no structural break for 1981 Q1.

	coefficient	std. error	prob.
Δd_t	0.3012	0.1331	0.0237
Δd_{t-1}	0.1622	0.1780	0.3621
Δd_{t-2}	-0.1605	0.1813	0.3760
Δd_{t-3}	-0.0598	0.1902	0.7533
Δd_{t-4}	-0.0441	0.1562	0.7776
Δd_{t-5}	0.1144	0.1847	0.5357
Δb_{t-1}	0.3534	0.0640	0.0000
σ_ν	0.0546	0.0019	0.0000

Table 3.11: Estimation results of coefficients in stock price and bubble equation, constant real interest rate, alternative dividend measure

Appendix 3.B Empirical results for the alternative dividend measure

This section presents the central results of the analysis using the alternative dividend series, U.S. net corporate dividend payments. It is seasonally adjusted. However, it contains more than the dividend payments on the S&P 500. We estimate

$$\Delta p_t = \sum_{i=0}^{5} \alpha_i \Delta d_{t-i} + \Delta b_t \tag{3.21}$$

$$\Delta b_t = \tau \Delta b_{t-1} + \nu_t \tag{3.22}$$

for a constant real interest rate and

$$\Delta p_t = \sum_{i=0}^{5} \alpha_i \Delta d_{t-i} + \sum_{j=0}^{7} \beta_j \Delta r_{t-j} + \Delta b_t \tag{3.23}$$

$$\Delta b_t = \tau \Delta b_{t-1} + \nu_t \tag{3.24}$$

for a time-varying real interest rate. Tables 3.11 and 3.12 show the estimation results of the state space model with and without a time-varying interest rate. The real interest rate is not significant in the estimation of stock prices; the coefficient on the lagged state variable, however, is significant. Tables 3.13 and 3.14 report the test results for the impact of the moral hazard indicators on the residuals of the state equation in levels and on the stock price in the measurement equation, respectively.

	coefficient	std. error	prob.
Δd_t	0.3388	0.1645	0.0395
Δd_{t-1}	0.1409	0.2217	0.5249
Δd_{t-2}	-0.1634	0.2757	0.5534
Δd_{t-3}	-0.0653	0.2089	0.7547
Δd_{t-4}	-0.0486	0.1965	0.8046
Δd_{t-5}	0.2057	0.2312	0.3736
Δr_t	0.8641	0.6653	0.1940
Δr_{t-1}	-0.4584	0.5447	0.4000
Δr_{t-2}	-0.3769	0.6006	0.5304
Δr_{t-3}	0.5318	0.4459	0.2330
Δr_{t-4}	-0.9838	0.6894	0.1536
Δr_{t-5}	0.0970	0.4994	0.8460
Δr_{t-6}	-0.4024	0.5802	0.4880
Δr_{t-7}	0.7548	0.6413	0.2392
Δb_{t-1}	0.3388	0.0744	0.0000
σ_ν	0.0535	0.0025	0.0000

Table 3.12: Estimation results of coefficients in stock price and bubble equation, time-varying real interest rate, alternative dividend measure

Test for misspecification of state equation

$$\hat{\eta}_t = \sum_{i=0}^{n} \theta_i x_{t-i} + \omega_t$$

time-varying real interest rate

indicator x_{t-i}	no. lags	lag i	coefficient	std. error	prob.
$\Delta \ln \xi_{t-i}$	0	0	-0.0016	0.0147	0.9124
$\Delta \ln m^{1987}_{t-i}$	0	0	0.0667	0.1599	0.6771
$\Delta \ln m^{1998}_{t-i}$	0	0	-0.0668	0.0697	0.3390
$\frac{Debt}{GDP}$ gap$_{t-i}$	3	1	-0.5530	0.8466	0.5144
		2	-0.4970	0.9965	0.6185
		3	0.7257	0.8237	0.3794
$\frac{Debt}{GDP}$ growth$_{t-i}$	6	1	0.2180	1.5058	0.8850
		2	0.5370	1.6976	0.7521
		3	0.2072	1.7648	0.9067
		4	-0.1266	1.7413	0.9421
		5	0.6819	1.6311	0.6763
		6	-1.0691	1.4242	0.4538

Table 3.13: Impact of moral hazard indicators on residuals from state equation in levels, alternative dividend measure

Augmented stock price equation

$$\Delta p_t = \sum_{i=0}^{5} \alpha_i \Delta d_{t-i} + \sum_{j=0}^{7} \beta_j \Delta r_{t-j} + \sum_{i=0}^{n} \theta_i x_{t-i} + \Delta b_t$$

indicator x_{t-i}	no. lags	lag i	coefficient	std. error	prob.
$\Delta \ln \xi_{t-i}$	0	0	−0.0001	0.0413	0.9974
$\Delta \ln m_{t-i}^{1987}$	0	0	−0.3622	0.3812	0.3421
$\Delta \ln m_{t-i}^{1998}$	0	0	−0.0709	0.2482	0.7751
$\frac{Debt}{GDP}$ gap$_{t-i}$	3	1	−0.4167	1.0737	0.6979
		2	0.2502	1.4529	0.8633
		3	−0.3448	1.2538	0.7833
$\frac{Debt}{GDP}$ growth$_{t-i}$	6	1	2.9552	1.7383	0.0891
		2	−0.3740	2.4246	0.8755
		3	−1.4031	2.0582	0.4954
		4	−0.2463	2.5758	0.9238
		5	0.0486	2.3060	0.9832
		6	−0.2078	2.0330	0.9186

Table 3.14: Impact of moral hazard indicators on stock prices in measurement equation, alternative dividend measure

Appendix 3.C Confidence bands for the estimated state variable with approximate estimation uncertainty

Lütkepohl (2005, p. 97) provides an approximation for the mean squared error (MSE) of the 1-step forecast with an estimated coefficient.

$$MSE\left(\Delta \hat{b}_t\right) = \frac{T+p+1}{T} MSE\left(\nu\right) \tag{3.25}$$

where T is the sample size and p the lag length, in our case $p = 1$. $MSE\left(\nu\right)$ are the mean squared errors from the estimated state equations (3.22) and (3.24), respectively. For $T \rightarrow \infty$, $MSE\left(\Delta \hat{b}_t\right) \rightarrow MSE\left(\nu\right)$. This approximation is derived for stationary processes. We use the more conservative measure of dividends, net corporate dividends. The results are presented in tables (3.11) and (3.12). Using Lütkepohl's approximation the estimation yields the following periods, during which there was a significant change in the bubble term as presented in tables (3.15) and (3.16).

Period	Sign of state variable
1955 Q3	+
1957 Q4	−
1961 Q1	+
1962 Q2	−
1970 Q2	−
1974 Q3	−
1975 Q1 - 1975 Q2	+
1980 Q3	+
1982 Q4	+
1987 Q1	+
1987 Q4	−
1997 Q3	+
1999 Q1	+
2002 Q3	−
2005 Q4	−

Table 3.15: Significant changes in the bubble term, constant real interest rate, approximated 95%-confidence band

Period	Sign of state variable
1957 Q4	−
1958 Q4	+
1961 Q1	+
1962 Q2	−
1969 Q3	−
1970 Q2	−
1974 Q2 - 1974 Q4	−
1975 Q1	+
1982 Q4	+
1987 Q1	+
1987 Q4	−
1999 Q1	+
2002 Q3	−
2005 Q4	−

Table 3.16: Significant changes in the bubble term, time-varying real interest rate, approximated 95%-confidence band

The results indicate a number of periods where the bubble grew or shrank significantly. In particular, the bubble grew in 1987 Q1 and shrank in 1987 Q4, which captures the events around Black Monday. Furthermore, the bubble grew in 1999 Q1 and shrank in 2002 Q3, which broadly corresponds to the results derived in section 6.

Chapter 4

The cyclicality of aggregate bank lending under bank capital regulation

4.1 Introduction

One of the most prominent measures of banking regulation is the minimum capital requirement, which states that a bank has to hold at least a certain fraction of its risk-weighted assets in equity as a buffer against insolvency. It is one of three core regulatory instruments of the Basel Capital Accord (Basel I), on which the Group of Ten (G10) countries have agreed and which entered into force in 1988. The other two pillars are an enhanced supervisory process and disclosure requirements about banks' risk profiles. Basel I was originally intended for the G10 countries, but meanwhile it has been incorporated into EU legislation as an EU capital requirements directive. On top of that a growing number of additional countries have adopted these rules (Jackson et al., 1999). Under the Basel Accord the minimum capital requirement is set to 8% and risk-weights are assigned according to borrower category (sovereign, bank or corporate entity). This categorisation has been critizised for its incentives for regulatory arbitrage: It doesn't differentiate between different degrees of risk among borrowers within one category. Banks will therefore tend to shift their portfolios towards investing in the relatively riskier projects within each category. To better align individual credit risk and the assigned risk-weights the Basel I framework has been revised. The New Basel Capital Accord (Basel II), which came into force in 2006, makes risk-weights contingent on borrower specific ratings. Risk-weights are calculated using borrowers specific ratings. Two types of ratings are allowed: The standard approach, where ratings on borrowers are supplied by external rating agencies, and an internal ratings-based (IRB) approach, where banks are allowed to produce their own measure of borrowers' riskiness, subject to their method being approved by the regulator. Under the IRB approach a bank uses its own estimates of key inputs to calculate risk-weights: The probability of default (PD) of a borrower, the loss given default (LGD), the exposure at default (EAD) and a maturity adjustment. Disclosure requirements under the New Basel Accord together with a periodic review of supervisors are aimed at ensuring that banks apply risk-weights that are consistent with their current business environment[1].

Yet many critics have argued that the new framework focuses too much on capital adequacy at the individual bank level which might lead to increased volatility of credit supply[2] in the aggregate exacerbating the business cycle, which is often referred to as procyclicality (Danielsson et al., 2001). It is argued that in a recession all types of loans become riskier and will be assigned a higher risk weight. This will lead to a fall in the capital adequacy ratio at all banks, which will then reduce their loan supply assuming that outside capital is difficult to raise in a recession. In the aggregate this will then lead to a worsening of the recession if firms have no other means of financing.

It is necessary to point out that loan supply without any regulation can be procyclical in itself, i.e. banks grant more loans in a boom than in a recession. This paper deals with the question whether the introduction of variable risk-weights under the Basel II Accord will make aggregate lending excessively procyclical compared to a situation with no capital requirement at all and to a minimum capital requirement with fixed risk-weights (Basel I). While it is certainly true that an increase in the risk-weights on the loan portfolio of an individual bank whose capital constraint is binding leads to a reduction in the bank's loan supply, it is not immediately clear what happens in the aggregate when only a fraction of banks in the economy is constrained. Given the observed excess capital holdings at many banks it is expected that not all banks will reduce lending in response to a given negative macroeconomic shock because their capital ratios have fallen below the required minimum.

To justify our approach observe from table 4.1 that these large EU banks hold more capital than the required minimum of 8%, i.e. a capital buffer, and that there is cross-sectional variation in the buffers. Different banks hold different capital ratios. In addition, Jokipii and Milne (2007) report average capital buffers across European countries ranging from 1.46%-points in the UK to 6.99%-points in Malta[3]. Furthermore, Peura and Jokivuolle (2004) report a median of

[1] For detailed explanations of the review process and the different methods for calculating risk-weights see Basel Committee on Banking Supervision (2005, 2006).

[2] We will use the terms credit supply, loan supply and (bank) lending interchangeably.

[3] Average values across banks (weighted by market share) in each country from 1997 to 2004. Capital buffers are total risk-weighted capital less the required minimum in each country (at least 8%). In most countries these buffers have no or a postive trend.

the total capital ratio of 11.2% for the US, Europe and Japan[4], which is well above the required 8%.

Bank name	Ratio of capital to risk-weighted assets
Commerzbank, D	12.6
Société Générale, F*	11.7
ABN AMRO Bank, NL	11.3
Barclays, UK	11.5
Deutsche Bank, D	13.6
HVB, D	10.4
Fortis, B	12.3
ING Bank, NL	11.5
Royal Bank of Scotland, UK	11.7
Rabobank Group, NL*	10.9
UBS, CH	13.6
HSBC, UK	12.0
HBOS, UK	11.8

Table 4.1: Capital adequacy ratios of large European banks
Notes: Numbers in percentage points as of 31.12.2004. An asterisk denotes data as of 31.12.2003. Source: Banks' annual reports and www.thebanker.com

The innovation in this paper is to allow for heterogeneity among banks with regard to their capital holdings and to explicitly model an interbank market, where banks can lend and borrow to attain their optimal loan supply. The question is then what happens when the economy faces an increase in aggregate risk, e.g. due to a downswing or recession, with the associated increase in risk-weights on all bank loans under Basel II. We measure excess credit volatility by comparing the response of aggregate lending to a change in macroeconomic risk with and without a capital constraint. Aggregate loan supply can only fluctuate to the extent that the liability side of the aggregate bank balance sheet fluctuates. In order to focus the analysis on the question whether risk-weights that vary according to aggregate risk have the capacity to induce excessive fluctuations in aggregate credit over and above the fluctuation by bank capital or debt, we hold the level of bank capital and bank debt constant such that the only source of cyclicality is changing macro risk[5]. However, if the liability side of the aggregate bank balance sheet is unchanged, no fluctuation in aggregate credit is possible. Merely the distribution of lending across different groups of banks might change. Therefore to allow for the possibility of aggregate credit fluctuation over and above the fluctuation of aggregate bank capital or bank debt one needs to allow for another asset that can be added or withdrawn from banks' balance sheet. As

[4] Bank level time-series averages over the period 1997 to 2001.

[5] An entirely different line of argument would be based on feedback effects from reduced lending via reduced repayments on bank debt to again reduced bank capital.

an example for an additional asset we use interbank loans. We then show that the procyclicality of aggregate bank lending depends crucially on the elasticity of supply of and demand for interbank loans, which in turn is reflected in the sensitivity of the interbank rate to a change in supply of and demand for it. We illustrate this mechanism by looking at an interbank market in which a central bank is able to withdraw or inject additional funds, thereby controlling the sensitivity of the interbank rate to changes in supply of and demand for it. The interest rate on loans to firms is determined by the opportunity and refinancing cost of lending to firms, which in the model is the interbank rate. If the additional supply of interbank funds by the constrained banks after an increase in aggregate risk is completely absorbed the interbank rate remains unchanged. It follows that lending by unconstrained banks doesn't change and aggregate lending is reduced. In contrast, if the interbank rate falls, opportunity costs of unconstrained banks fall and their lending increases, thereby offsetting partly or fully the reduction in lending by constrained banks.

Our main findings are that the degree of excess procyclicality depends on the sensitivity of the interbank rate to a change in aggregate risk, on the sensitivity of loan supply to firms with regard to a change in the opportunity costs, the sensitivity of risk-weights to a change in aggregate risk and on the proportion of constrained banks in the economy after an increase in aggregate risk. Note that the analysis is purely positive and takes regulation as given while asking what are the implications for fluctuations in aggregate credit.

An important question is whether the observed capital buffers under the Basel I regime with constant risk-weights can be expected to exist under the New Basel Accord with variable risk-weights. After all the objective of the reform of the Basel regulatory framework is to better align regulatory and economic bank capital. Economic bank capital is in this context the captial ratio a bank would optimally choose to hold in the absence of regulatory requirements. Regulatory capital might be the socially optimal level of capital. A financial safetey net e.g. might reduce the incentives for an individual bank to hold enough capital. A minimum capital requirement might be desirable to bring up banks' capital holdings towards the socially desirable level. Perfectly aligning regulatory and economic bank capital could mean that there will be no buffers. However, there are a number of reasons for why banks might nevertheless hold more capital than required even with variable risk-weights. Banks might hold a capital buffer as financial slack to be able to exploit unexpected profit opportunities; furthermore, banks might want to insure against financial distress which might arise from the consequences of violating the minimum capital requirement and the subsequent regulatory penalties and associated reputational loss (Berger, Herring and Szegö, 1995). In addition, banks are required to hold more than the minimum under the supervisory arrangements of pillar 2 of the New Basel Accord (Basel Committee on Banking Supervision, 2006). Moreover, Lowe (2002) argues that the disclosure requirements of pillar 3 of the New Basel Accord are likely to lead to capital holdings in excess of the required minimum because a future need to raise additional capital when the capital ratio falls might be anticipated by capital markets already in good times. In sum, the introduction of capital requirements itself might change the level of economic capital.

The remainder of the paper is organised as follows: Section 2 relates our paper to the literature, section 3 sets up the model and presents solutions for aggregate credit supply and its response to a macroeconomic shock with and without the capital constraint; section 4 assesses the degree of excess procyclicality on the basis of the model implications, section 5 discusses some features of the model, and section 6 concludes.

4.2 Related literature

A number of studies have looked at the problem of procyclicality under bank capital regulation. Allen and Saunders (2004) look at the sources of cyclical variations in the elementary inputs to calculating risk-weights, however without working with a model. In addition, Lowe (2002) discusses the influence of aggregate risk on the measurement of risk-weights and looks at the possible macroeconomic consequences, however also without a formal model. Both papers come to the conclusion that the proposed methods for calculating risk-weights under Basel II are indeed likely to lead to cyclical variations in risk-weights. Furthermore, there are papers that use models to evaluate the effect of a minimum capital requirement on aggregate credit supply. Catarineu-Rabell, Jackson and Tsomocos (2005) evaluate the likelihood of procyclical credit supply in a calibrated model with a capital constraint à la Basel II. Their focus is on the optimal choice of the method to provide internal ratings of borrowers. Specifically, they look at three scenarios of procyclical, countercyclical and constant borrowers' rating quality. They find that without regulation banks would opt for a countercyclical rating method, while they would choose the procyclical one if regulation forbids the countercyclical one. Blum and Hellwig (1995) look at a macroeconomic model with banks, into which they introduce a capital constraint with constant risk-weights. They don't consider distributional effects of bank capital and conclude that bank capital regulation will reinforce the procyclicality of bank lending, investment and therefore output. The source of cyclicality in their model is cyclical bank capital, which in our model is constant. Also in their model the constraint is binding either for all banks or for none. Estrella (2004) uses a dynamic model of an optimising representative bank, which trades off the costs of holding and adjusting capital versus the costs of default to yield an optimal level of capital holdings. He arrives at the conclusion that risk-sensitive capital requirements under Basel II might give rise to procyclical capital requirements and bank lending. Kashyap and Stein (2004) look at the optimal regulatory capital requirement. The regulator optimises the trade-off between allowing banks to issue loans efficiently and ensuring systemic stability. As a result they propose state-contingent capital requirements, which decrease in a downswing. They show in a calibrated model that otherwise there might be quite large fluctuations in credit supply. Hofmann (2005) shows with a calibrated model of credit portfolio risk that binding capital constraints increase the fluctuation of credit supply. Our paper differs from those in that we allow banks to hold more capital than required and that banks hold different amounts. Similarly, Heid (2007) explicitly takes into account the existence of capital buffers. The reason why the bank holds a capital buffer is that the owners might incur a loss in case of default of the bank, which is larger than the costs of holding a capital buffer. However, his model also uses a representative bank and there is no interbank market. His conclusion is that capital buffers can mitigate the extent of procyclicality. Also, Repullo and Suarez (2007) use a representative bank to evaluate procyclical credit supply. Their conclusion is that although banks may choose to hold a capital buffer a larger contraction of credit supply in a recession under Basel II than under Basel I is likely.

Tanaka (2001) analyses the effect of bank capital regulation on the monetary transmission mechanism and finds that on top of making loan supply overly sensitive to a change in macro risk, it weakens the power of monetary policy to stimulate the economy when the capital constraint becomes binding. Chen (2001) shows in a dynamic model that a bank capital requirement together with a firm collateral requirement produces amplified credit volatility. In Chen's model banks are implicitly capital constraint because with too little capital banks can't commit to monitoring and can't raise sufficient deposits. They have to reduce lending instead. Goodhart and others have in various studies (Goodhart, Hofmann and Segoviano, 2004; Goodhart, 2005) argued that risk-sensitive bank capital regulation will be more procyclical than with constant risk-weights. However, in the same studies it is mentioned that banks might want to hold a capital buffer above the required minimum and this might act as an offsetting factor. Also, Caruana (2005) argues that enhanced risk-management systems under pillar 2 will make a bank more forward-looking and thus better prepared for times of trouble enabling it to react in a timely and adequate fashion to avoid sharp cuts in credit supply.

Another related branch in the literature deals with the interaction of a minimum bank capital requirement and monetary policy (von Peter, 2004; Chami and Cosimano, 2001; Zicchino, 2005; Cecchetti and Li, 2005). Overall, however, these models either look at a representative bank or at the aggregate bank balance sheet and neglect distributional aspects.

There is also a large strand of the literature that deals with modelling an interbank market, e.g. Agion, Bolton and Dewatripont (1999), Freixas and Parigi (1998), Rochet and Tirole (1996), Allen and Gale (2000) or Acharya (2001). Typically, however, these models are concerned with modelling different structures of an interbank market and the implications for systemic risk via domino effects or contagion. They do not look at the implications for macroeconomic variables like aggregate lending or output. Similarly, a framework set up by Eichberger and Summer (2005) looks at mutual credit exposure in the interbank market and assesses the implications for systemic stability. In what follows we will start from their framework because it is suitable to incorporate heterogeneous banks in a tractable way. We adapt it to look at the implications of capital adequacy requirements for aggregate loan supply. In their model they only briefly mention that the effects on aggregate lending are unclear. We provide a detailed analysis of just this point and are able to work out the exact mechanism by which the distribution and aggregate volume of bank lending is affected.

4.3 The model

The model builds on the framework of a banking system developed by Eichberger and Summer (2005). There, firms have a binary investment opportunity of fixed size such that changes in aggregate credit come about indirectly by credit rationing of firms. In contrast we employ a continuous loan demand function derived from firm profit maximisation along the lines of Gray and Wu (1995) to analyse the cyclical behaviour of credit supply.

4.3.1 Loan demand

There is a number of firms in each sector i of the economy, each of which has access to a neoclassical production technology. Loan demand by firms is derived from a standard profit maximisation problem. Each firm buys capital at the price of 1 using a bank loan L_i and produces output y_i under decreasing marginal returns to capital using a production function.

$$y_i = sqL_i^{\alpha}$$

where $\alpha < 1$. Firms in each sector i are distinguished by an individual productivity parameter q, which is ex-ante unknown to both the firm and the bank. It is randomly assigned after receiving the loan and is uniformly distributed over $[0, M]$. Banks offer the same interest rate to all firms from one sector since they can't observe individual productivities when making the loan. $s = \{0, 1\}$ is an aggregate shock capturing firms' success or failure. If $s = 0$ there is no output in any sector and no repayment of any outstanding loan by firms. The probability that $s = 1$ is given by

$$\Pr(s = 1) = \rho$$

with $0 < \rho < 1$. The higher ρ the better the state of the economy and the lower is aggregate risk. The interest rate R_i on bank loans to firms in sector i is taken as given by each firm. A firm will apply for a loan only if the payoff from producing and repaying the loan is positive, given that the aggregate shock $s = 1$.

$$qL_i^\alpha - R_i L_i \geq 0$$

where the payoff to the outside option is normalized to zero. Define q^*as $q^* L_i^\alpha - R_i L_i = 0$

$$q^* = R_i L_i^{1-\alpha} \tag{4.1}$$

For values of q below q^* the firm makes a negative profit and defaults on its credit liabilities while for values above q^* the firm succeeds and pays back its loan. The critical value of the productivity shock is higher with a higher interest rate and with a larger loan size, i.e. a high interest rate or a large loan size make a default more likely. The expected profit of each firm is then given by

$$E(\pi^{firm}) = \rho \int_{q^*}^{M} (qL_i^\alpha - R_i L_i) g(q) dq$$

where $g(q)$ is the probability density function of q. Using the uniform distribution of q, (4.1) and maximising with respect to L_i yields the optimal loan demand, which falls with a higher interest rate.

$$L_i^d = \left(\frac{\alpha}{2-\alpha} \frac{M}{R_i} \right)^{\frac{1}{1-\alpha}} \tag{4.2}$$

4.3.2 Optimal loan supply without regulation

4.3.2.1 Individual bank's loan supply

We assume that there is one bank in each sector of the economy. Banks are relationship lenders. Each bank in the economy has an informational advantage over other banks in lending to its long-term customer firms. Therefore each firm belongs to the customer group of exactly one bank and can only borrow from that bank. As a consequence banks act as monopolists when lending to their customers. Moreover, since ex-ante all firms are identical from the point of view of the bank and all firms in one sector depend on one single bank, the index i also refers to bank i.

In our model risk-neutral banks differ in their holdings of capital. Bank capital e_i is assumed to be distributed according to some distribution function, with density $f(e_i)$ and cumulative distribution function $F(e_i)$. This might be because they have different future profit opportunities as they serve different sectors of the economy. When maximising profits they take into account their discounted value from running a banking business, the charter value (Keeley, 1990; Hellmann, Murdock and Stiglitz, 2000). Thus, banks with a higher charter value hold a larger capital buffer than banks with a lower charter value[6].

It is assumed that capital is difficult to raise in the short run such that banks take it as given in their decision on loan supply. Banks maximise their expected return from making loans to firms and other banks taking into account interest payments on deposits, equity and loans taken from the interbank market.

There is another important assumption to make. Firms mustn't be able to substitute bank finance by other means of financing (Kashyap and Stein, 1994). Otherwise investment and production wouldn't depend on bank loan supply.

Each bank has a given amount of deposits d_0, which is assumed to be the same for all banks, and a given amount of equity e_i, which varies across banks, available to make loans to firms. It is assumed that in the short-run bank equity is fix. With a given amount of debt for all banks balance sheets only differ in equity capital. This assumption allows to concentrate entirely on the impact of variable risk-weights together with differences in capital holding on the cyclicality of bank lending and implies that in this model well capitalised banks are those that have a relatively large amount of funds available for lending. However, it also implies that ceteris paribus there can only be fluctutations in aggregate credit to the extent that the amount of interbank funds in the system varies[7]. The bank can lend or borrow in the interbank market at a competitively determined interbank rate. Following Eichberger and Summer (2005) we define $l^+ = \max\{l_i; 0\}$ and $l^- = -\min\{l_i; 0\}$ to denote an interbank lender's and borrower's position, respectively, where l_i denotes an interbank loan. The interbank rate is r_I, the return to equity r_E and the deposit rate is normalized to zero.

[6] Since we have a static model, however, this is not explicitly modelled. For a formal model refer to Elizalde and Repullo (2007).

[7] This is one possibility to allow for fluctuations of the aggregate bank balance sheet. It could also be accomplished by any other additional asset that can be added to or withdrawn from the banking system.

The payoff from investment for bank i is

$$\pi^{bank} = \int_{q*}^{M} s_i R_i L_i g(q) dq - r_E e_i - r_I l_i^- + \delta r_I l_i^+$$

where δ is the discount on the return from interbank loans that is due to some banks defaulting on their interbank loans[8]. Note that in this framework interbank loans are assumed to be settled via a central clearing house without any direct bilateral exposure among banks. The aggregate shock s_i introduces the possibility for banks to go bankrupt. Bank i's expected payoff is given by

$$E(\pi^{bank}) = \rho \int_{q*}^{M} R_i L_i g(q) dq - r_E e_i - r_I l_i^- + \bar{\delta} r_I l_i^+$$

where $\bar{\delta}$ is the expected discount on the interbank return due to defaults. To produce a benchmark case to which we can compare the solution with the capital constraint we first derive the optimal loan supply without a capital constraint by maximising bank i's expected profit subject to the loan demand function, the budget constraint and two non-negativity constraints for the interbank positions.

$$\begin{gathered} \max_{\{R_i, l_i\}} E(\pi^{bank}) \\ s.t. \\ L_i = \left(\tfrac{\alpha}{2-\alpha}\tfrac{M}{R_i}\right)^{\frac{1}{1-\alpha}} \\ L_i + l_i^+ \leq e_i + d_0 + l_i^- \\ l_i^+ \geq 0 \\ l_i^- \geq 0 \end{gathered}$$

The result is an optimal loan supply function

$$L_i = \begin{cases} \left(\frac{\rho MB}{r_I}\right)^{\frac{1}{1-\alpha}} & if \qquad e_i \leq \left(\frac{\rho MB}{r_I}\right)^{\frac{1}{1-\alpha}} - d_0 \\ e_i + d_0 & if \quad \left(\frac{\rho MAB}{r_I}\right)^{\frac{1}{1-\alpha}} - d_0 \leq e_i \leq \left(\frac{\rho MB}{\bar{\delta} r_I}\right)^{\frac{1}{1-\alpha}} - d_0 \\ \left(\frac{\rho MB}{\bar{\delta} r_I}\right)^{\frac{1}{1-\alpha}} & if \qquad e_i \geq \left(\frac{\rho MB}{\bar{\delta} r_I}\right)^{\frac{1}{1-\alpha}} - d_0 \end{cases} \tag{4.3}$$

where $B = \frac{\alpha^2(2-2\alpha)}{(2-\alpha)^2}$. The optimal loan supply to firms by banks equals the

[8]Note that neither the firm nor the bank gets anything in case of default on the part of the firm. It makes the model easier to solve and doesn't change the qualitative implications. An interpretation could be that in case of a firm default the liquidation value the bank as the creditor can get is zero. Or else that the bank has to pay a fraction of the firm's production it can recover as auditing costs. Then assume that this fraction is one (Bernanke, Gertler and Gilchrist, 1998).

optimal loan demand by firms. Thus (4.3) is the equilibrium in the market for bank loans. The optimal bank loan supply, however, differs across banks according to the size of the liability side of their balance sheet, which is here uniquely determined by the amount of capital a bank holds. Moreover, the optimal bank loan supply is determined by the interbank rate, which is the refinancing cost of lending to firms for interbank borrowers and the opportunity cost of lending to firms for interbank lenders. Given the assumption of a fixed size of debt for all banks, low capitalised banks have a lower optimal loan supply than well capitalised ones, the difference of which is due to the difference in refinancing and opportunity cost of lending to firms. For low capitalised banks the interbank rate r_I is the refinancing cost, whereas for well capitalised banks the interbank rate r_I times the discount factor $\overline{\delta}$ is the opportunity cost. The first line shows the optimal loan supply by interbank borrowers. Banks whose optimal loan supply exceeds available funds borrow in the interbank market. Their holdings of equity are too low to cover the desired amount of lending to firms at their refinancing cost r_I. The second line shows the optimal loan supply by all banks whose available funds exceed desired loan supply at the refinancing cost of r_I yet fall short of the desired lending at the opportunity cost $\overline{\delta}r_I$. These banks are considered to be inactive in the interbank market and adjust their interest rate instead to match demand for their loans to their available funds. Thus, the separating force into interbank borrowers and lenders is the discount on the return in the interbank market. The third line shows the optimal loan supply of those banks whose available funds exceed their desired lending at their respective opportunity costs $\overline{\delta}r_I$. Banks with available funds exeeding their optimal loan supply lend the difference in the interbank market.

For interbank borrowers and lenders, the optimal loan supply to firms decreases with aggregate risk, i.e. increases with the success probability ρ of the customer pool, with the range M of the distribution of the individual productivity parameter and decreases with the opportunity and refinancing costs in the interbank market, r_I and $\overline{\delta}r_I$ respectively. Since well capitalised banks offer lower interest rates, their optimal loan supply is larger but also the volatility of their optimal loan supply. In this unregulated system loan supply is procyclical in the sense that it decreases with aggregate risk since loan supply for all three groups of banks varies positively with ρ. The question later on will be whether the degree of procyclicality under regulation exceeds the one without regulation.

4.3.2.2 The interbank market and determinants of $\overline{\delta}$

Banks lend to and borrow from a central clearing house in the interbank market and there are no bilateral interbank exposures. Instead the central clearing house channels funds from interbank lenders to interbank borrowers. Eichberger and

Summer (2005) argue that this captures an anonymous competitive interbank market. A central clearing mechanism allows to abstract from a risk-adjusted bilateral interbank rate and is consistent with a perfectly competitive interbank market, where the interbank rate is determined by aggregate supply of and demand for interbank loans. The interbank rate is determined in a competitive equilibrium in the interbank market, where demand for interbank funds equals their supply.

$$\int_i l_i^- = \int_i l_i^+$$

Interbank borrowers pay the competitively determined interbank rate r_I and interbank lenders receive $\bar{\delta} r_I$ in expected terms. In the model by Eichberger and Summer (2005) $\bar{\delta}$ can be derived by assuming each interbank lender gets an equal share of available repayments by interbank borrowers if repayments fall short of claims due to the default of some banks. In this model, however, $s = \{0, 1\}$ for all banks. The expected discount on interbank loans $\bar{\delta}$ equals the probability of success of a bank's portfolio, which in turn is equal to the probability of success in production in each sector.

$$E(\delta) = \bar{\delta} = E(s_i) = \rho$$

As such interbank loans appear as risky as loans to firms from the point of view of an individual bank. However, in practice loans to other banks are deemed safer than loans to firms due to an explicit or implicit government guarantee for interbank loans, which is due to the role the interbank market plays for financial stability and the associated incentives for the government not to let a large number of banks fail. In this model these considerations are not derived endogenously, rather it is assumed that interbank loans are less risky than loans to firms. However, this assumption is not crucial in this context since the existence of a functioning interbank market requires that not all banks be constrained at once. From this follows that only the interbank borrower banks can be constrained because they are the ones with little equity in the model. These banks only have interbank liabilities on their balance sheet, which are not assigned any risk-weights. Therefore, in the model, it is not crucial what risk-weights interbank assets carry.

4.3.2.3 Aggregate loan supply without regulation

Summing over all banks in (4.3) yields aggregate loan supply by all banks without regulation L^U.

$$L^U = aF(a - d_0) + \int_{a-d_0}^{b-d_0} (e_i + d_0) f(e_i)\, de_i + b[1 - F(b - d_0)] \tag{4.4}$$

where

$$\begin{aligned} a &= \left(\frac{\rho MB}{r_I}\right)^{\frac{1}{1-\alpha}} \\ b &= \left(\frac{MB}{r_I}\right)^{\frac{1}{1-\alpha}} \end{aligned}$$

where a is the optimal loan supply to firms by banks which are interbank borrowers, $F(a - d_0)$ is the proportion of interbank borrowers, b is the optimal loan supply to firms by banks which lend in the interbank market and $1 - F(b - d_0)$ is their proportion. The proportion of banks that are interbank borrowers, lenders or not active in the interbank market is entirely determined by banks' holdings of capital because debt is assumed to be the same for all banks. As stated in the beginning, these are simplifying assumptions to be able to exclusively focus on the role of variable risk-weights for the cyclical behaviour of aggregate loan supply. Figure 4.1 illustrates aggregate loan supply to firms across banks with different capital holdings. In the upper part, banks with capital below $a - d_0$ are interbank borrowers because their available funds $e_i + d_0$ fall short of desired lending a at the refinancing cost r_I. Interbank borrowing for each of these banks is the diffence between a and $e_i + d_0$, depicted by the triangular l_i^-. Banks with capital above $b - d_0$ are interbank lenders because their available funds exceed desired lending b at the opportunity cost $\bar{\delta} r_I$. Interbank loans for each of these banks are the difference between $e_i + d_0$ and b, depicted by the triangular l_i^+. Banks with capital above $a - d_0$ but below $b - d_0$ are not active in the interbank market and lend the sum of their capital and debt, $e_i + d_0$, to firms. In the lower part a hypothetical cumulative distribution function of bank capital is graphed, from which the proportions of each type of bank can be read off.

4.3.3 Optimal loan supply under regulation

4.3.3.1 Loan supply by constrained banks

We now impose a capital constraint on the banking system. Regulation requires banks to hold at least a fraction c of risk weighted assets wL_i as capital e_i.

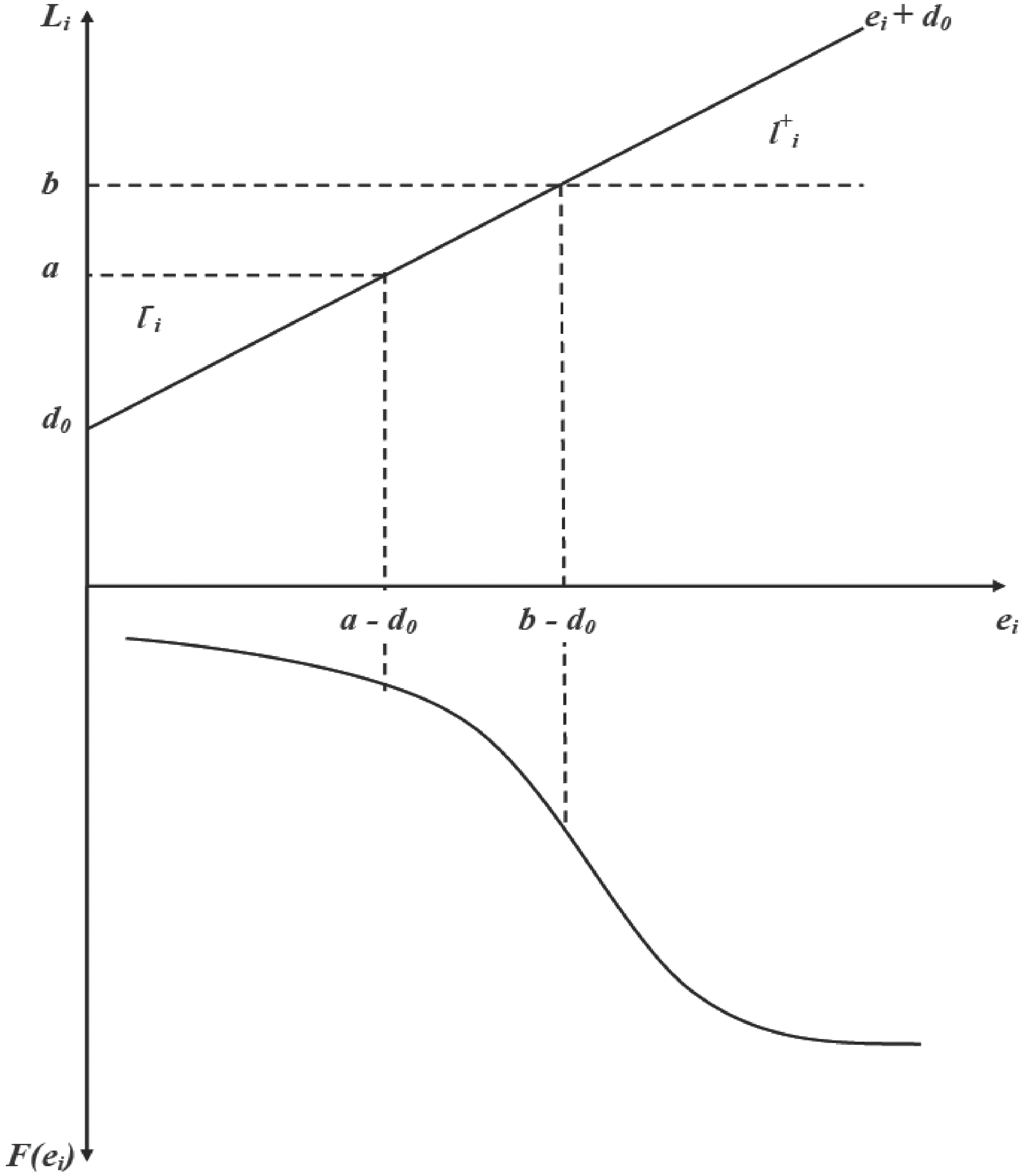

Figure 4.1: Aggregate loan supply to firms across banks without a bank capital constraint

Risk-weights w depend on aggregate risk with

$$w = w(\rho)$$
$$w' = \frac{\partial w(\rho)}{\partial \rho} < 0$$

Risk-weights depend only on aggregate risk, more specifically the probability that production in each sector of the economy is positive. In practice, the risk-weights also depend on borrower specific characteristics. We abstract from these, however, to focus on the presumption that as the economy experiences a downswing or recession all types of borrowers appear riskier than before and receive a higher risk-weight. Moreover, the idiosyncratic risk is diversifiable by banks. Under the Basel I Accord risk-weights were constant, whereas the innovation in the revised framework, Basel II, is to make risk-weights dependent on borrower riskiness, part of which is aggregate risk. Precisely this innovation sparked the debate about potential procyclical aggregate credit supply. The capital constraint can be written as

$$e_i \geq cw(\rho) L_i$$

where c is some value fixed by the regulator. From bank profit maximisation one can show that the capital constraint is binding if

$$\begin{aligned} e_i &< \tilde{h} \\ \tilde{h} &\equiv cw(\rho)\left(\frac{\rho MB}{\tilde{r}_I}\right)^{\frac{1}{1-\alpha}} = cw(\rho)\,\tilde{a} \end{aligned} \tag{4.5}$$

Variables with a tilde denote variables in the scenario with a capital constraint. The introduction of a capital constraint changes the equilibrium interbank rate and therefore the opportunity cost of lending to firms. Condition (4.5) says that banks with capital below $\tilde{h}$ are constrained from lending as much as they would like to. In addition the budget constraint is binding too. Also it is necessary to assume that there are some interbank borrowers left who are willing to absorb funds in the interbank market. Therefore we look at the case where the critical value $\tilde{h}$ does not exceed the threshold at which a bank ceases to borrow in the interbank market. As a consequence, in the model, only interbank borrowers are potentially constrained and interbank lenders are always unconstrained. Optimal loan supply by constrained banks L_i^c is then given by

$$L_i^c = \frac{e_i}{cw(\rho)} \quad if \quad e_i < \tilde{h} \tag{4.6}$$

Loan supply is determined by the capital constraint and is equal to a multiple $\frac{1}{cw(\rho)}$ of capital. Furthermore, the cyclical properties depend on the sensitivity of risk-weights w with respect to aggregate risk. As ρ rises risk-weights fall and loan supply rises. The more risk-weights react to changes in ρ the stronger is the effect on loan supply by constrained banks. Unconstrained banks under regulation behave essentially as in the unregulated case, except that under regulation they might face a different interbank rate.

4.3.3.2 Aggregate loan supply under regulation

Summing over all constrained banks as well as the remaining unconstrained banks yields the aggregate loan supply under regulation L^R.

$$\begin{aligned} L^R &= \frac{1}{cw(\rho)} \int_0^{\tilde{h}} e_i f(e_i)\, de_i + \tilde{a}\left[F(\tilde{a}-d_0) - F\left(\tilde{h}\right)\right] \\ &\quad + \int_{\tilde{a}-d_0}^{\tilde{b}-d_0} (e_i + d_0) f(e_i)\, de_i + \tilde{b}\left[1 - F\left(\tilde{b}-d_0\right)\right] \end{aligned}$$

The first term is loan supply to firms by constrained banks with capital below $\tilde{h}$, the second term is loan supply to firms by the remaining unconstrained interbank borrowers. The third and fourth term are lending by banks not active in the interbank market and by interbank lenders. Figure 4.2 illustrates the situation after the introduction of bank capital regulation.The fairly steep upward sloping line is loan supply to firms by constrained banks[9]. Banks with capital below or equal to $\tilde{h}$ supply loans according to a multiple $\frac{1}{cw(\rho)}$ of their capital. Among these banks some of them are interbank lenders because their (constrained) lending to firms is lower than their available funds $e_i + d_0$. Conversely, some constrained banks are interbank borrowers because their (constrained) lending exceeds available funds $e_i + d_0$. Finally, by assumption a part of interbank borrowers remain unconstrained. These banks have capital above $\tilde{h}$ but below $\tilde{a}$. Note that the introduction of bank capital regulation changes the proportion of interbank borrowers and lenders. Therefore the equilibrium interbank rate is changed under a capital constraint. Comparing figures 4.1 and 4.2 one can notice that desired interbank borrowing is smaller under regulation than without regulation, given the same distribution function for bank capital[10]. Therefore the interbank rate should be lower with a capital constraint, $\tilde{r}_I < r_I$. Intuitively the

[9] Its slope is larger than 1 for reasonable values for c and $w(\rho)$. Under the Basel Accord $c = 0.08$ such that for risk weights up to 12.5 the slope is larger than one.

[10] For now the distribution of bank capital is held fixed. In a later section the effects of a changing distribution of bank capital will be discussed.

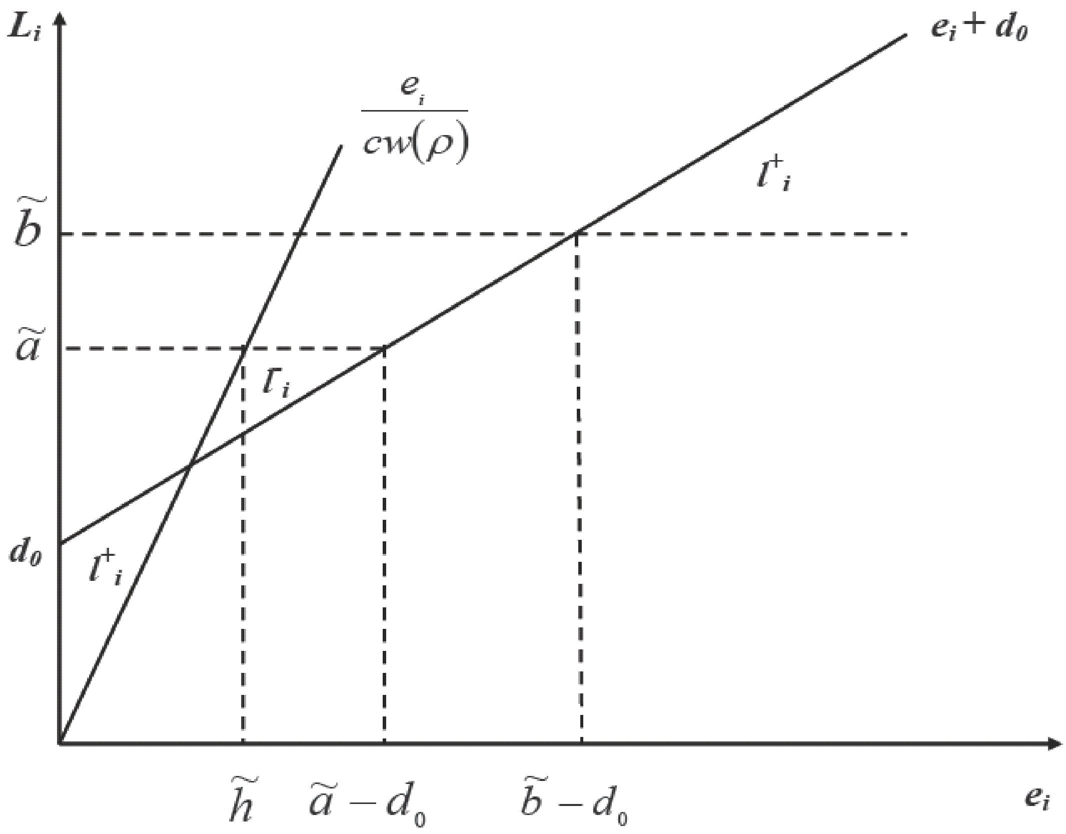

Figure 4.2: Aggregate loan supply to firms with a bank capital constraint

reason is that some banks that originally borrowed from the interbank market are forced by the capital constraint to borrow less or even lend funds in the interbank market. As a result lending to firms by interbank borrowers and lenders is larger under regulation, $\tilde{a} > a$ and $\tilde{b} > b$.

4.4 Assessing procyclicality

4.4.1 Fluctuations of aggregate lending without a capital constraint

To derive a benchmark for cyclicality of aggregate lending to which we can compare the cyclicality under bank capital regulation consider an increase in the probability of success ρ, i.e. a decrease in aggregate risk, in the situation without a capital requirement. One could interpret this as an upswing or boom in the economy. From (4.4) it is clear that interbank borrowers want to increase their loan supply if ρ rises because expected profits of banks increase with a higher probability of success in each sector. Note however, that interbank lenders don't increase lending. The reason is that at the same time as the probability of success of firms rises so does the expected repayment from lending in the interbank market, since $\bar{\delta} = \rho$, such that the opportunity cost of lending to firms increases to the same extent as the expected profit from lending to firms increases. This means that demand for interbank loans rises while supply stays the same such that the interbank rate rises.

$$\frac{dr_I}{d\rho} > 0$$

The appendix provides an exact solution for the response of the interbank rate to a rise in ρ. Therefore in deriving an analytical expression for the response of aggregate loan supply we need to take the reaction of the interbank rate to a change in aggregate risk into account. The first derivative with respect to aggregate risk ρ yields

$$\frac{\partial L^U}{\partial \rho} = a' F\left(a - d_0\right) + b'\left[1 - F\left(b - d_0\right)\right] \tag{4.7}$$

where $F(e_i)$ is the cumulative distribution function of bank capital, $r_I' = \frac{dr_I}{d\rho}$ is the change in the interbank rate after a change in ρ, and

$$a' = \frac{1}{1-\alpha}\frac{MB\left(r_I - \rho r_I'\right)}{r_I^2}\left(\frac{\rho MB}{r_I}\right)^{\frac{\alpha}{1-\alpha}} \tag{4.8}$$

$$b' = -\frac{1}{1-\alpha}\frac{MBr_I'}{r_I^2}\left(\frac{MB}{r_I}\right)^{\frac{\alpha}{1-\alpha}} \tag{4.9}$$

The change in aggregate lending in response to a change in aggregate risk is the sum of the marginal response of lending by interbank borrowers times their proportion and the marginal change in lending by interbank lenders times their proportion. The marginal response of loan supply to firms by interbank borrowers is positive, since $0 < \frac{dr_I}{d\rho} < \frac{r_I}{\rho}$[11]. In contrast the marginal response of interbank lenders is negative. Intuitively this is because for interbank lenders the induced credit expansion to firms due to an increase in the probability of success ρ is offset by a corresponding increase in the expected return from interbank lending due to the same increase in ρ, which affects interbank lender banks via their counterparts in the interbank market. In addition, desired lending to firms and therefore desired borrowing from the interbank market increases for interbank borrowers, which tends to increase the interbank rate. As a result loans to firms by interbank lenders tend to decrease ($b' < 0$) and loans to firms by interbank borrowers tend to increase ($a' > 0$).

However, since bank capital and bank debt are fixed for all banks there can be no change in the aggregate volume of lending to firms. Therefore the increase in lending to firms by interbank borrowers is exactly offset by the decrease in

[11] See appendix for proof.

lending by interbank lenders. Merely the distribution of lending to firms across different banks changes.

In order to allow for fluctuations in aggregate lending over and above fluctuations in aggregate bank capital and debt, there needs to be an asset that can be added or withdrawn from banks' aggregate balance sheet. This could be any asset like bank capital, bank debt, government bonds or interbank funds. To focus on the role of variable risk-weights in the fluctuation of aggregate credit, bank capital needs to be held constant. In any other case, the ease with which this additional asset could be sold or bought determines the extent to which aggregate lending can fluctuate. A central bank which is able to reduce or increase the aggregate amount of interbank funds by open market interventions is a convenient example to illustrate how aggregate fluctuations in lending to firms depend on the sensitivity of the interest rate on alternative assets. It is convenient because it avoids the introduction of another asset, while yielding the same insights. However, since there is no inflation in the model the central bank is really just an example. What matters is the elasticity of supply and demand of any outside asset.

Example: A central bank Consider a central bank which intervenes in the interbank market to inject or withdraw funds with the aim of minimising a typical loss function. The loss function is increasing in the deviation of inflation and output from target. Moreover, the central bank places a certain weight on the output gap versus the deviation of inflation from target. Output and inflation both fall with the interest rate, which is in the model equal to the interbank rate. Consider a cost-push shock which pushes up output and decreases inflation. The weight on output vs. inflation determines the strength of an interest rate response to a cost-push shock. In the model an increase in the probability of success ρ could be the result of a cost-push shock. The elasticity of demand for or supply of interbank funds by the central bank is reflected in the sensitivity of the interbank rate to a change in ρ. In the one extreme case were the central bank only cares about inflation it might intervene in the interbank market to reduce the interbank rate by supplying additional funds. Then both interbank borrowers and lenders increase loans to firms after the increase in ρ. Consequently, aggregate lending to firms increases. The other extreme case the central bank doesn't place any weight on inflation and only on the output gap and doesn't intervene at all, is equivalent to the original situation, in which aggregate lending doesn't change at all because the interbank rate adjusts to keep aggregate lending and therefore output constant.

In sum, the cyclical behaviour of aggregate lending crucially depends in the model on the sensitivity of the interbank rate with respect to changes in aggregate risk. More precisely the sensitivity of the interbank rate is determined by the elasticity of supply of and demand for alternative assets to firm credits, which in the model are interbank loans. Therefore the sensitivity of the interbank rate in response to a change in ρ proxies for the ease with which outside funds are added to or withdrawn from the banking system. In the following sections fluctuations of aggregate credit with and without a capital constraint have therefore to be compared for a given degree of sensitivity of the interbank rate to changes in ρ.

4.4.2 Fluctuations of aggregate lending with a capital constraint

The response of aggregate lending under regulation to a change in aggregate risk ρ is given by

$$\begin{aligned} \frac{\partial L^R}{\partial \rho} &= \tilde{a}' \left[F\left(\tilde{a} - d_0\right) - F\left(\tilde{h}\right) \right] + \tilde{b}' \left[1 - F\left(\tilde{b} - d_0\right) \right] \\ &\quad - \frac{w'}{cw\left(\rho\right)} \int_0^{\tilde{h}} e_i f\left(e_i\right) de_i \end{aligned} \tag{4.10}$$

where again $F(e_i)$ is the cumulative distribution function of bank capital, $\tilde{a}' = a'\left(\tilde{r}_I\right)$ and $\tilde{b}' = b'\left(\tilde{r}_I\right)$ are the marginal responses of interbank borrowers and lenders, evaluated at the lower interbank rate under regulation $\tilde{r}_I$, and $\tilde{a} > a$ and $\tilde{b} > b$ are loan supply to firms by interbank borrowers and lenders, respectively[12]. The response of aggregate lending is composed of the marginal response of unconstrained interbank borrowers times their proportion, the marginal response of interbank lenders times their proportion and the marginal response of constrained banks times their proportion. Note that the marginal response of constrained banks depends on the sensitivity of risk-weights with respect to aggregate risk. For $c \longrightarrow 0$ the expression collapses into the one for the unregulated system.

4.4.3 A measure of procyclicality

To analyse the effect of a capital constraint on the cyclical behaviour of aggregate bank lending we look at the difference Δ in the marginal change in lending to

[12] Since the marginal response of the interbank rate to a change in ρ is taken to be given and the same under both regimes it is denoted by r_I' throughout.

firms after a change in ρ with and without the capital constraint.

$$\Delta = \frac{\partial L^R}{\partial \rho} - \frac{\partial L^U}{\partial \rho}$$

In the following we wish to analyse the determinants of Δ. We are thus interested in the degree of excess procyclicality and whether Δ could potentially become negative. There are three scenarios that we would like to compare: no capital constraint, a capital constraint with constant risk-weights (Basel I) and a capital constraint with variable risk-weights (Basel II). Denote the difference in fluctuation without the constraint and a constraint with constant risk-weights by Δ^I, and the difference in fluctuation without the constraint and a constraint with variable risk-weights by Δ^{II}.

4.4.4 Constant risk-weights

A situation with constant risk-weights corresponds to the regulatory framework of Basel I. Since we only look at the case where bank capital is fixed Δ^I is

$$\begin{aligned} \Delta^I = \; & -\tilde{a}' F\left(\tilde{h}\right) \\ & + \tilde{a}' F\left(\tilde{a} - d_0\right) + \tilde{b}' \left[1 - F\left(\tilde{b} - d_0\right)\right] \\ & - a' F\left(a - d_0\right) - b' \left[1 - F\left(b - d_0\right)\right] \end{aligned} \tag{4.11}$$

where a', b', $\tilde{a}'$, $\tilde{b}'$ and $\tilde{h}$ are defined as above. The first line is the avoided fluctuation of lending to firms by constrained banks, whose lending doesn't vary with ρ. Fluctuations in lending by constrained banks can only be driven by fluctuations in their bank capital, which is ruled out in the model. The second line is the sum of the marginal responses of lending by interbank borrowers and lenders under a minimum capital requirement times their proportion respectively. The third line is the same measure without any capital constraint. The difference between the second and third line is that the interbank rate is lower under regulation, which has an impact on the proportions of interbank borrowers and lenders and on their marginal responses to ρ.

The degree of procyclicality depends on the change in proportions of interbank borrowers and lenders due to the lower interbank rate after introduction of a capital requirement, and their respective sensitivities of lending to a change in ρ. The reason is that the introduction of capital regulation changes the equilibrium interbank rate and therefore the marginal responses of lending to firms to a change in ρ. Moreover the marginal responses of lending to firms also depend on the elasticity of the interbank rate with respect to ρ as can be seen from (4.8) and (4.9). Then the degree of excess procyclicality depends on whether the response of lending of unconstrained banks under a regulated regime is more or less sensitive to ρ than their response in an unregulated regime. Moreover, since desired unconstrained lending also changes with regulation because the opportunity cost r_I changes, the proportion of interbank borrowers and lenders changes under regulation too.

More specifically, since $\tilde{r}_I < r_I$ the proportion of unconstrained interbank borrowers, $F(\tilde{a} - d_0)$, is higher under regulation because the lower interbank rate makes it profitable for more banks to lend to firms. These additional banks will now act as borrowers in the interbank market, whereas before they belonged to the group of banks inactive in the interbank market. Similarly, the proportion of interbank lenders, $1 - F\left(\tilde{b} - d_0\right)$, will fall because some of them now find it less profitable to lend in the interbank market at the lower interest rate $\tilde{r}_I$.

The size of the marginal responses of lending to firms by these two types of banks with and without a capital requirement, depends crucially on the sensitivity of the interbank rate to a change in aggregate risk ρ as can be seen from (4.8) and (4.9). In particular the marginal response of interbank lenders under regulation, $\tilde{b}'$, is always larger in absolute terms than without regulation, b', for a given value of r_I'. Thus, there are two countervailing effects with regard to interbank lenders: On the one hand, they tend to reduce their lending because $\tilde{r}_I$ rises after an increase in ρ. On the other hand their proportion in the economy falls as the threshold rises above which a bank is an interbank lender.

Whether the marginal response of lending to firms by interbank borrowers is larger or smaller under regulation depends on the sensitivity of the interbank rate to a change in risk, r_I'. For values of $r_I' < k\frac{\tilde{r}_I}{\rho}$, where $k < 1$, $\tilde{a}' > a'$, i.e. as long as the sensitivity of the interbank rate with respect to aggregate risk is not too large, the marginal response of lending to firms by unconstrained interbank borrower banks is larger under regulation than without regulation[13]. The reason is that in response to a decrease in aggregate risk (ρ rises) banks want to increase their lending. However, at the same time the increase in ρ also increases the interbank rate r_I because more banks want to borrow in the interbank market and fewer want to lend there. The introduction of a capital requirement increases the first effect of the two because the marginal response of lending to a change in ρ is larger with a lower interbank rate $\tilde{r}_I$, while the second one is taken as determined exogenously (e.g. by a central bank). Therefore there exists a value for r_I' below which the marginal response of lending to firms by interbank borrowers to a change in ρ is larger under regulation than without.

In sum there are a number of countervailing effects that jointly determine the extent of (excess) procyclicality under a regime of bank capital regulation à la Basel I, i.e. with fixed risk-weights.

[13]See appendix for a derivation of k.

4.4.5 Variable risk-weights

The situation with variable risk-weights corresponds to the revised Basel framework, Basel II, and is the source of concern over excessively procyclical behaviour of aggregate bank lending. To evaluate this concern look at the difference in credit fluctuation without a capital constraint and one with variable risk-weights Δ^{II}.

$$\begin{aligned}\Delta^{II} &= -\tilde{a}'F\left(\tilde{h}\right)-\frac{w'}{cw\left(\rho\right)}\int_{0}^{\tilde{h}}e_{i}f\left(e_{i}\right)de_{i}\\&\quad+\tilde{a}'F\left(\tilde{a}-d_{0}\right)+\tilde{b}'\left[1-F\left(\tilde{b}-d_{0}\right)\right]\\&\quad-a'F\left(a-d_{0}\right)-b'\left[1-F\left(b-d_{0}\right)\right]\end{aligned}$$

Use $\Delta^{I}$and Δ^{II} to find the additional fluctuation in credit induced by the introduction of variable risk-weights into existing bank capital regulation.

$$\Delta^{II}-\Delta^{I}=-\frac{w'}{cw\left(\rho\right)}\int_{0}^{\tilde{h}}e_{i}f\left(e_{i}\right)de_{i}>0 \tag{4.12}$$

This shows that there is indeed reason for concern about variable risk-weights increasing the fluctuation of aggregate credit. There are three qualifications to make, though. The first is that the extent of excess procyclicality clearly depends on the sensitivity of risk-weights with respect to changes in $w'\left(\rho\right)$. The smaller it is, the less excess procyclicality will occur. Second, the degree of excess procyclicality also depends on the share of constrained banks, holding capital of less than $\tilde{h}$. The smaller this share the smaller excess procylicality. Third, from (4.11) the question arises whether aggregate credit is unambiguously excessively procyclical under a capital constraint with constant risk-weights compared to the unregulated system. This brings up the question whether the introduction of bank capital regulation has the potential to even reduce the fluctuations of aggregate credit supply.

4.4.6 Can bank capital regulation reduce credit fluctuations?

To answer that question look at (4.11) again and note that $\Delta^{I}<0$ if

$$\tilde{a}'\left[F\left(\tilde{a}-d_{0}\right)-F\left(\tilde{h}\right)\right]+\tilde{b}'\left[1-F\left(\tilde{b}-d_{0}\right)\right]<a'F\left(a-d_{0}\right)+b'\left[1-F\left(b-d_{0}\right)\right]$$

Suppose, first, that the equilibrium interbank rate r_{I} doesn't change upon introduction of a capital requirement. In the example with a central bank this could be the case when the central bank absorbes the additional supply of interbank funds by those banks that are now constrained, accommodating the resulting drop in aggregate bank lending. Then, $\Delta^I < 0$ if

$$-\tilde{a}'F\left(\tilde{h}\right) < 0$$

a condition which is fulfilled if $r_I' < \frac{r_I}{\rho}$. Again there are two effects from a rise in ρ on the marginal response of lending $\tilde{a}'$: The first comes from the fact that a higher ρ makes lending to firms more attractive and tends to increase loan supply to firms. The second works via increasing the interbank rate and thereby offsetting the tendency to expand loan supply. As long as the second effect is not too large, loan supply by interbank borrower banks increases. Thus, these banks would expand lending to firms but are constrained from doing so because their capital is below $\tilde{h}$. Instead their lending is determined by (4.6), which doesn't vary with ρ. Therefore the degree of a change in aggregate lending in response to a change in ρ is reduced.

Second, suppose that the interbank rate decreases from r_I to $\tilde{r}_I$ upon introduction of the capital constraint as before. Then, $\Delta^I < 0$ if, first, $k\frac{\tilde{r}_I}{\rho} < r_I' < \frac{\tilde{r}_I}{\rho}$ and, second, the proportions of unconstrained interbank borrowers and lenders don't change very much after imposing the capital requirement. From condition one follows that the marginal responses of lending to firms by interbank lenders are negative, with the response under regulation being larger in absolute terms than without regulation, $\tilde{b}' < b' < 0$. Moreover, the marginal response of lending to firms by interbank borrowers under regulation is positive but smaller than without regulation, $\tilde{a}' < a'$. If additionally, the proportions of unconstrained interbank borrowers and lenders with and without the capital requirement don't change very much, it is possible that the degree of procyclicality is even reduced under a capital requirement. Whether condition two is met depends essentially on the shape of the distribution function of bank capital, for a given difference between r_I and $\tilde{r}_I$[14]. These are examples that excessively procyclical aggregate credit is not a necessary consequence of bank capital regulation in this model.

4.5 Discussion

The distribution of bank capital The introduction of a bank capital constraint is aimed at ensuring adequate capitalisation of banks in order to increase financial stability. Initially constrained banks might aim at restoring their bank capital to attain the optimal level of lending to firms in the absence of the capital

[14]The appendix provides a detailed illustration of this point.

restriction. Therefore it might be expected that the distribution of bank capital changes with the introduction of a stringent capital requirement. In the model that means that the proportion of banks that hold equity below the required minimum decreases. In (4.12) that would lead to a decrease in excess procyclicality when switching from Basel I to Basel II, since the proportion of banks holding less capital than $\tilde{h}$ decreases.

With regard to the cyclicality of bank lending under a Basel I regime with constant risk-weights, the proportion of banks with capital less than $\tilde{h}$ also decreases. At the same time, however, the equilibrium interbank rate will be affected since the ratio of interbank borrowers and lenders might change. To the extent that banks will raise their capital to a level not much above the required one, the proportion of interbank borrowers increases again, which tends to increase the interbank rate again.

Cyclical bank capital holdings It is plausible to allow bank capital to vary over the business cycle too. In a downswing as more firms default on their loans banks write them off their equity holdings. Conversely in an upswing bank profits increase and might add to their capital holdings. In the present analysis we have abstracted from this aspect to fully concentrate on the effect of the introduction of variable risk-weights. However, to briefly comment on the impact of cyclical bank capital holdings, note that they would introduce some cyclicality of the aggregate banking balance sheet over and above the one induced by adding or withdrawing interbank funds. Therefore the interbank rate needn't adjust as much in response to a change in aggregate risk. However, even with cyclical capital holdings the need for an additional asset that can be added or withdrawn from the aggregate banking balance sheet remains to yield an effect on the fluctuation of aggregate bank lending over and above the fluctuation in aggregate bank capital holdings.

4.6 Conclusion

Starting from the observation that most banks hold more capital than the required minimum and moreover to varying extents we have set up a model of a banking system in which we can analyse the effect of a change in aggregate risk, like in an upswing or in a downswing, on aggregate credit supply to firms. We find that the extent to which aggregate bank credit fluctuates with or without a capital constraint depends crucially on the ease with which additional assets like funds from the interbank market can be added to or withdrawn from the aggregate balance sheet of the banking sector, which is reflected in the sensitivity of the interbank rate with respect to a change in aggregate risk. In addition, the introduction of a bank capital constraint changes the equilibrium interbank rate, the marginal responses of bank lending to a change in risk and the proportions of interbank borrowers and lenders. Therefore the effect of introducing a bank capital constraint with constant or with variable risk-weights affects not only the behaviour of constrained but also the one of the unconstrained banks. Consequently, the cyclicality of aggregate credit is a function of the sensitivity of risk-weights with respect to changes in risk, the responses of lending by constrained and by unconstrained banks in the system, their respective shares in the banking sector and the sensitivity of the interbank rate with regard to a change in aggregate risk.

References

Acharya, V. (2001): A theory of systemic risk and design of prudential bank regulation. Mimeo, New York University.

Aghion, P., P. Bolton and M. Dewatripont (1999): Contagious bank failures in a free banking system. *European Economic Review* 44, 713-718.

Allen, F. and D. Gale (2000): Financial contagion. *Journal of Political Economy,* Vol. 108, No. 1, 1-33.

Allen, L. and A. Saunders (2004): Incorporating systemic influences into risk measurements: a survey of the literature. *Journal of Financial Services Research* 26 (2), 161-192.

Basel Committee on Banking Supervision (2005): An explanatory note on the Basel II IRB risk weight functions. Bank for International Settlements.

Basel Committee on Banking Supervision (2006): International Convergence of Capital Measurement and Capital Standards: A Revised Framework - Comprehensive Version. Bank for International Settlements.

Berger, A., R. Herring and G. Szegö (1995): The role of capital in financial institutions. Wharton Working Paper 95-01.

Bernanke, B., M. Gertler and S. Gilchrist (1998): The financial accelerator in a quantitative business cycle framework. National Bureau of Economic Research, Working Paper No. 6455.

Blum, J. and M. Hellwig (1995): The macroeconomic implications of capital adequacy requirements for banks. *European Economic Review* 89, 739-749.

Caruana, J. (2005): Monetary policy and Basel II. 33rd Economics Conference, Vienna, 13 May 2005.

Caterineu-Rabell, E., P. Jackson and D. Tsomocos (2005): Procyclicality and the new Basel Accord - banks' choice of loan rating system. *Economic Theory* 26, 537-557.

Cecchetti, S. and L. Li (2005): Do capital adequacy requirements matter for monetary policy? National Bureau of Economic Research, Working Paper No. 11830.

Chami, R. and T. Cosimano (2001): Monetary policy with a touch of Basel. International Monetary Fund Working Paper No. WP/01/151.

Chen, N. (2001): Bank net worth, asset prices and economic activity. *Journal of Monetary Economics* 48, 415-436.

Danielsson, J., P. Embrechts, C. Goodhart, C. Keating, F. Muennich, O. Renault and H. Shin (2001): An academic response to Basel II. London School of Economics Financial Markets Group and ESRC Research Centre, Special Paper No. 130.

Eichberger, J. and M. Summer (2005): Bank capital, liquidity and systemic risk. *Journal of the European Economic Association,* Vol. 3, April-May, 547-555.

Elizalde, A. and R. Repullo (2007): Economic and regulatory capital in banking: What is the difference? *International Journal of Central Banking,* Vol. 3, No. 3, 87-117.

Estrella, A. (2004): The cyclical behaviour of optimal bank capital. *Journal of Banking and Finance* 28, 1469-1498.

Freixas, X. and B. Parigi (1998): Contagion and efficiency in gross and net payment systems. *Journal of Financial Intermediation* 7, 3-31.

Goodhart, C., B. Hofmann and M. Segoviano (2004): Bank regulation and macroeconomic fluctuations. *Oxford Review of Economic Policy,* Vol. 20 No. 4, 591-615.

Goodhart, C. (2005): Financial regulation, credit risk and financial stability. *National Institute Economic Review,* No. 192, April, 118-127.

Gray, J. and Y. Wu (1995): On equilibrium credit rationing and interest rates. *Journal of Macroeconomics,* Vol. 17, No. 3, 405-420.

Heid, F. (2007): The cyclical effects of the Basel II capital requirements. *Journal of Banking and Finance* 31, 3885-3900.

Hellmann, T., K. Murdock and J. Stiglitz (2000): Liberalization, moral hazard in banking, and prudential regulation: Are capital requirements enough? *American Economic Review,* Vol. 90, No. 1,147-165.

Hofmann, B. (2005): Procyclicality: the macroeconomic impact of risk-based capital requirements. *Financial Markets and Portfolio Management,* Vol. 19, No. 2, 179-200.

Jokipii, T. and A. Milne (2007): The cyclical behaviour of European bank capital buffers. Research report No. 56, Swedish Insitute for Financial Research.

Keeley, M. (1990): Deposit insurance, risk and market power in banking. *American Economic Review,* Vol. 80, No. 5, 1183-1200.

Lowe, P. (2002): Credit risk measurement and procyclicality. Bank for International Settlements Working Paper 116.

Peura, S. and E. Jokivuolle (2007): Simulation based stress tests of banks' regulatory capital adequacy. *Journal of Banking and Finance* 28, 1801-1824.

Kashyap, A. and J. Stein (1994): Monetary policy and the bank lending channel, in: Mankiw, G. (ed.): *Monetary policy*, National Bureau of Economic Research.

Kashyap, A. and J. Stein (2004): Cyclical implications of the Basel II capital standards. Economic Perspectives Q1, Federal Reserve Bank of Chicago.

Repullo, R. and J. Suarez (2007): The procyclical effects of Basel II. Mimeo, Centro de Estudios Monetarios y Financieros.

Rochet, J. and J. Tirole (1996): Interbank lending and systemic risk. *Journal of Money, Credit and Banking*, Vol. 28, No. 4, Part 2: Payment Systems Research and Public Policy Risk, Efficiency, and Innovation, 733-762.

Tanaka, M. (2001): How do bank capital and capital adequacy regulation affect the monetary transmission mechanism? CESifo Working Paper No. 799.

Von Peter, G. (2004): Asset prices and banking distress: a macroeconomic approach. Bank for International Settlements Working Paper No. 167.

Zicchino, L. (2005): A model of bank capital, lending and the macroeconomy: Basel I versus Basel II. Bank of England Working Paper No. 270.

Appendix 4.A Derivation of firms' loan demand

The firm in sector i maximises

$$\begin{aligned} E(\pi^{firm}) &= \rho \int_{q^*}^{M} (qL_i^\alpha - R_i L_i) g(q) dq \\ &= \rho \left(\frac{1}{2} M L_i^\alpha - R_i L_i + \frac{R_i^2 L_i^{2-\alpha}}{2M} \right) \end{aligned}$$

taking R_i as given and $q^* = R_i L_i^{1-\alpha}$. This yields the first order condition

$$\frac{\alpha}{2} M - R_i L_i^{1-\alpha} + (2-\alpha) \frac{R_i^2}{2M} L_i^{2-2\alpha} = 0$$

which is quadratic in L_i with two solutions

$$L_i^1 = \left(\frac{\alpha}{2-\alpha} \frac{M}{R_i} \right)^{\frac{1}{1-\alpha}}$$

and

$$L_i^2 = \left(\frac{M}{R_i} \right)^{\frac{1}{1-\alpha}}$$

We choose the first solution because under the second one the threshold q^* for a positive expected payoff would be at the maximum M and loan demand by firms would be zero.

Appendix 4.B Optimal loan supply without regulation

Bank i's problem is

$$\begin{gathered} \max_{\{R_i, l_i\}} E(\pi^{bank}) \\ s.t. \\ L_i = \left(\tfrac{\alpha}{2-\alpha} \tfrac{M}{R_i} \right)^{\frac{1}{1-\alpha}} \\ L_i + l_i^+ \leq e_i + d_0 + l_i^- \\ l_i^+ \geq 0 \\ l_i^- \geq 0 \end{gathered}$$

which results in the Lagrangean

$$\begin{aligned}
\mathfrak{L} &= \rho\frac{2-2\alpha}{2-\alpha}\left(\frac{\alpha}{2-\alpha}\frac{M}{R_i}\right)^{\frac{1}{1-\alpha}} R_i - r_E e_i - r_I l_i^- + \bar{\delta} r_I l_i^+ \\
&\quad + \lambda(e_i + d_0 + l_i^- - \left(\frac{\alpha}{2-\alpha}\frac{M}{R_i}\right)^{\frac{1}{1-\alpha}} - l_i^+) + \tau l_i^+ + \sigma l_i^-
\end{aligned}$$

with the Langrangean multipliers λ, τ, σ.

The first order conditions are:

$$\rho\alpha\frac{2-2\alpha}{2-\alpha} - \lambda R_i^{-1} = 0 \tag{4.13}$$

$$\bar{\delta} r_I - \lambda + \tau = 0 \tag{4.14}$$

$$-r_I + \lambda + \sigma = 0 \tag{4.15}$$

$$\lambda(e_i + d_0 + l_i^- - \left(\frac{\alpha}{2-\alpha}\frac{M}{R_i}\right)^{\frac{1}{1-\alpha}} - l_i^+) = 0 \tag{4.16}$$

$$\tau l_i^+ = 0 \tag{4.17}$$

$$\sigma l_i^- = 0 \tag{4.18}$$

From (4.14) $\lambda > 0$. Thus, three cases remain to be looked at. Also note that from (4.14) and (4.15) that $\tau + \sigma > 0$, i.e. a bank will never lend and borrow in the interbank market at the same time.

Case 1: $\sigma = 0$ **and** $\tau > 0$
$l_i^+ = 0$ and $l_i^- = \left(\frac{\alpha}{2-\alpha}\frac{M}{R_i}\right)^{\frac{1}{1-\alpha}} - e_i - d_0 \geq 0$. The bank will borrow from the interbank market. By (4.14) and (4.15) $r_I = \lambda$, and by (4.13)

$$\begin{aligned}
R_i &= \frac{r_I(2-\alpha)}{\rho\alpha(2-2\alpha)} \\
L_i^* &= \left(\frac{\rho M B}{r_I}\right)^{\frac{1}{1-\alpha}}
\end{aligned}$$

where $B = \frac{\alpha^2(2-2\alpha)}{(2-\alpha)^2}$. This case applies if

$$e_i \leq \left(\frac{\rho M B}{r_I}\right)^{\frac{1}{1-\alpha}} - d_0$$

Case 2: $\sigma > 0$ **and** $\tau > 0$
$l_i^+ = l_i^- = 0$. The bank will neither lend nor borrow from the interbank market. From (4.14) and (4.15) $\delta r_I < \lambda < r_I$. From the budget constraint we get

$$R_i^* = \frac{\alpha M}{(2-\alpha)(e_i + d_0)^{1-\alpha}}$$

$$L_i^* = e_i + d_0$$

And this case applies for

$$\left(\frac{\rho MB}{r_I}\right)^{\frac{1}{1-\alpha}} - d_0 < e_i < \left(\frac{\rho MB}{\bar{\delta} r_I}\right)^{\frac{1}{1-\alpha}} - d_0$$

Case 3: $\sigma > 0$ **and** $\tau = 0$

$l^- = 0$ and $l^+ = e_i + d_0 - \left(\frac{M}{R_i}\right)^{\frac{1}{1-\alpha}} \geq 0$. The bank is an interbank lender. The optimal interest rate is

$$R_i^* = \frac{\bar{\delta} r_I (2-\alpha)}{\rho\alpha(2-2\alpha)}$$

and the optimal loan size is

$$L_i^* = \left(\frac{\rho MB}{\bar{\delta} r_I}\right)^{\frac{1}{1-\alpha}}$$

This case applies for

$$e_i > \left(\frac{\rho MB}{\bar{\delta} r_I}\right)^{\frac{1}{1-\alpha}} - d_0$$

Appendix 4.C Optimal loan supply under regulation

The bank's problem is the same as without regulation only subject to an additional constraint.

$$\begin{gathered}
\max_{\{R_i, l_i\}} E(\pi^{bank}) \\
s.t. \\
L_i = \left(\tfrac{\alpha}{2-\alpha}\tfrac{M}{R_i}\right)^{\frac{1}{1-\alpha}} \\
L_i + l_i^+ \leq e_i + d_0 + l_i^- \\
l_i^+ \geq 0 \\
l_i^- \geq 0 \\
e_i \geq cw(\rho) L_i
\end{gathered}$$

The resulting Lagrangean is given by

$$
\begin{aligned}
\mathfrak{L} &= \rho\frac{2-2\alpha}{2-\alpha}R_i\left(\frac{\alpha}{2-\alpha}\frac{M}{R_i}\right)^{\frac{1}{1-\alpha}} - r_E e_i - \tilde{r}_I l_i^- + \bar{\delta} r_I l_i^+ \\
&+\lambda(e_i + d_0 + l_i^- - \left(\frac{\alpha}{2-\alpha}\frac{M}{R_i}\right)^{\frac{1}{1-\alpha}} - l_i^+) + \tau l_i^+ + \sigma l_i^- \\
&+\mu\left(e_i - cw\left(\frac{\alpha}{2-\alpha}\frac{M}{R_i}\right)^{\frac{1}{1-\alpha}}\right)
\end{aligned}
$$

The first order conditions are

$$
\begin{aligned}
\rho\alpha\frac{2-2\alpha}{2-\alpha} - (\lambda + \mu cw)\, R_i^{-1} &= 0 && (4.19)\\
\bar{\delta}\tilde{r}_I - \lambda + \tau &= 0 && (4.20)\\
-\tilde{r}_I + \lambda + \sigma &= 0 && (4.21)\\
\lambda(e_i + d_0 + l_i^- - \left(\frac{\alpha}{2-\alpha}\frac{M}{R_i}\right)^{\frac{1}{1-\alpha}} - l_i^+) &= 0 && (4.22)\\
\tau l_i^+ &= 0 && (4.23)\\
\sigma l_i^- &= 0 && (4.24)\\
\mu\left(e_i - cw\left(\frac{\alpha}{2-\alpha}\frac{M}{R_i}\right)^{\frac{1}{1-\alpha}}\right) &= 0 && (4.25)
\end{aligned}
$$

$\lambda > 0$ for the same reasons as above. With $\mu = 0$ the capital constraint is not binding and the case without regulation applies. The capital constraint is binding if $\mu > 0$, and by (4.19) and (4.25) this applies if

$$
e_i < cw\left(\frac{\rho MB}{\lambda}\right)^{\frac{1}{1-\alpha}}
$$

Again there are three cases to distinguish: $\sigma > 0$ and $\tau = 0$, $\sigma = 0$ and $\tau > 0$, $\sigma = 0$ and $\tau = 0$ with the same implications as above. Note that in order to have a functioning interbank market $0 < r_I < \infty$. Otherwise there is either excess demand or supply and no equilibrium. Thus not all interbank borrowers must be constrained at once, which imposes the restriction:

$$
cw\left(\frac{\rho MB}{\tilde{r}_I}\right)^{\frac{1}{1-\alpha}} < \left(\frac{\rho MB}{\tilde{r}_I}\right)^{\frac{1}{1-\alpha}} - d_0
$$

or

$$
\frac{d_0}{1-cw} < \left(\frac{\rho MB}{\tilde{r}_I}\right)^{\frac{1}{1-\alpha}}
$$

Appendix 4.D The response of the equilibrium interbank rate to a change in aggregate risk

One can derive an expression for the response of r_I to a change in ρ, $\frac{\partial r_I}{\partial \rho}$ by taking the total derivative of the interbank equilibrium condition with respect to r_I and ρ.

$$\begin{aligned} \int\limits_i l_i^- &= \int\limits_i l_i^+ \\ \int\limits_0^{a-d_0} \left[a - (e_i + d_0)\right] f(e_i)\, de_i &= \int\limits_{b-d_0}^{e_{\max}} \left[e_i + d_0 - b\right] f(e_i)\, de_i \end{aligned}$$

where $e_{\max}$ is bank capital of the bank with the highest bank capital holdings. Implicit differentiation yields

$$\left[F(a-d_0)\frac{\partial a}{\partial r_I} + \frac{\partial b}{\partial r_I} - F(b-d_0)\frac{\partial b}{\partial r_I}\right] dr_I = -F(a-d_0)\frac{\partial a}{\partial \rho} d\rho$$

which results in

$$\frac{dr_I}{d\rho} = \frac{r_I}{\rho}\left(1 + \frac{\frac{\partial b}{\partial r_I}\left[1 - F(b-d_0)\right]}{\frac{\partial a}{\partial r_I} F(a-d_0)}\right)^{-1}$$

Together with

$$\begin{aligned} \frac{\partial a}{\partial r_I} &< 0 \\ \frac{\partial b}{\partial r_I} &< 0 \end{aligned}$$

it follows

$$0 < \frac{dr_I}{d\rho} < \frac{r_I}{\rho}$$

Appendix 4.E The size of the marginal responses of lending to firms with and without regulation

Consider the marginal response of lending to firms by interbank lenders as given by

$$b' = -\frac{1}{1-\alpha}\frac{MBr_I'}{r_I^2}\left(\frac{MB}{r_I}\right)^{\frac{\alpha}{1-\alpha}}$$

and

$$\tilde{b}' = -\frac{1}{1-\alpha}\frac{MBr_I'}{\tilde{r}_I^2}\left(\frac{MB}{\tilde{r}_I}\right)^{\frac{\alpha}{1-\alpha}}$$

respectively. Since $\tilde{r}_I < r_I$ under regulation and for a given value of r_I' it follows that

$$\left|\tilde{b}'\right| > |b'|$$

The marginal response of lending to firms by interbank borrowers without regulation as given by

$$a' = \frac{1}{1-\alpha}\frac{MB\left(r_I - \rho r_I'\right)}{r_I^2}\left(\frac{\rho MB}{r_I}\right)^{\frac{\alpha}{1-\alpha}}$$

is smaller than the corresponding measure under regulation

$$\tilde{a}' = \frac{1}{1-\alpha}\frac{MB\left(\tilde{r}_I - \rho r_I'\right)}{\tilde{r}_I^2}\left(\frac{\rho MB}{\tilde{r}_I}\right)^{\frac{\alpha}{1-\alpha}}$$

i.e. $\tilde{a}' > a'$, if

$$\frac{\partial r_I}{\partial \rho} < \frac{1-\left(\frac{\tilde{r}_I}{r_I}\right)^{\frac{1}{1-\alpha}}}{1-\left(\frac{\tilde{r}_I}{r_I}\right)^{\frac{2-\alpha}{1-\alpha}}}\frac{\tilde{r}_I}{\rho} \equiv k\frac{\tilde{r}_I}{\rho}$$

Appendix 4.F The role of the distribution function of bank capital

In section 4.4 it was shown that the degree of excess procyclicality depends on a number of different factors, one of which is the exact shape of the cumulative distribution function of bank capital. Figure 4.3 shows one theoretical distribution function $F(e_i)$ with the associated thresholds separating banks into interbank borrowers, $e_i < a - d_0$, and interbank lenders, $e_i > b - d_0$. The introduction of a minimum capital requirement reduces the equilibrium interbank rate r_I to $\tilde{r}_I$. This leads to an increase in both thresholds to $\tilde{a} - d_0$ and $\tilde{b} - d_0$. The associated change in the proportions of interbank borrowers and lenders depend on the exact shape of the distribution function. Figure 4.3 illustrates the case where the change in these proportions is small. For other shapes of the distribution function it might be much larger.

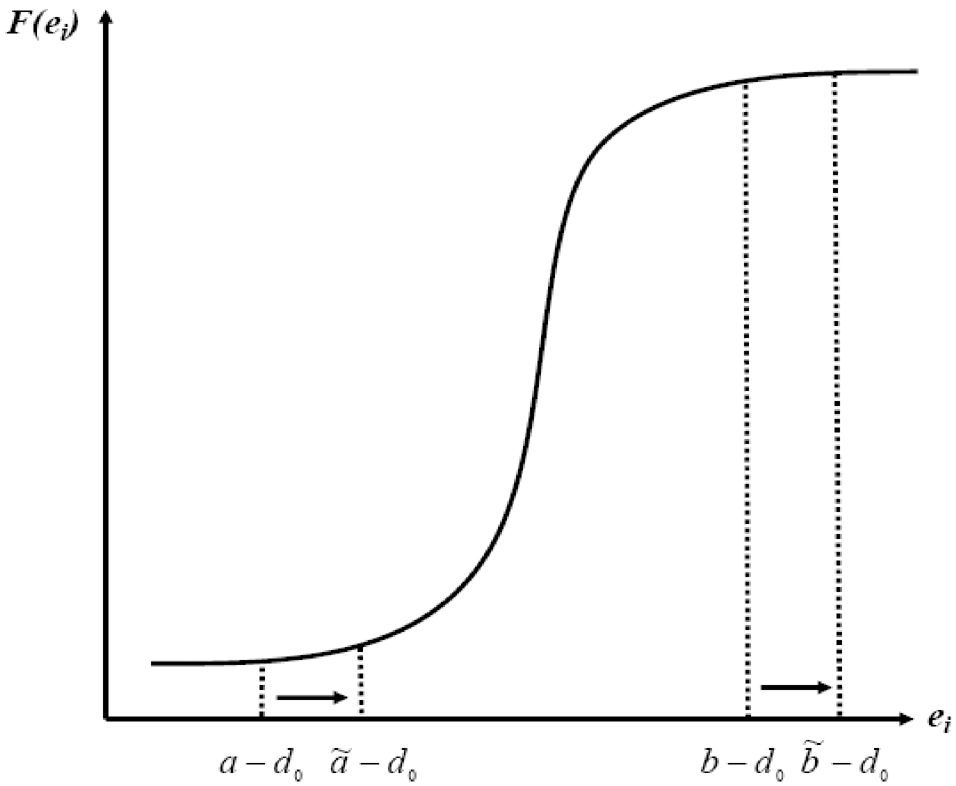

Figure 4.3: Theoretical cumulative distribution function of bank capital

Curriculum Vitae

October 2004 - July 2008	Research and teaching assistant Seminar für Makroökonomie, Ludwig-Maximilians-Universität, München
October 2004 - July 2008	Ph.D. programme in economics Munich Graduate School of Economics, Ludwig-Maximilians-Universität, München
Sept. 2003 - Sept. 2004	MSc Economics University of Warwick, UK
Sept. 2002 - June 2003	Exchange student University of Warwick, UK
October 2000 - August 2003	B.A. European Economic Studies Otto-Friedrich-Universität, Bamberg
1999	Abitur Neues Gymnasium Nürnberg